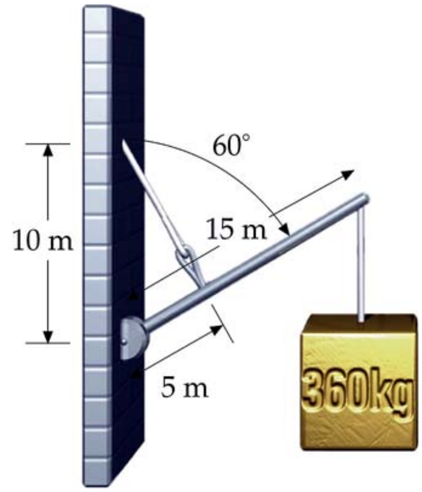
Physics 135-1 Midterm II, Spring 2017 **Name ___________________________**

1) A mass of 360 kg is hanging from a massless rod as shown at right. The rod is attached to a frictionless pivot at its lower end.



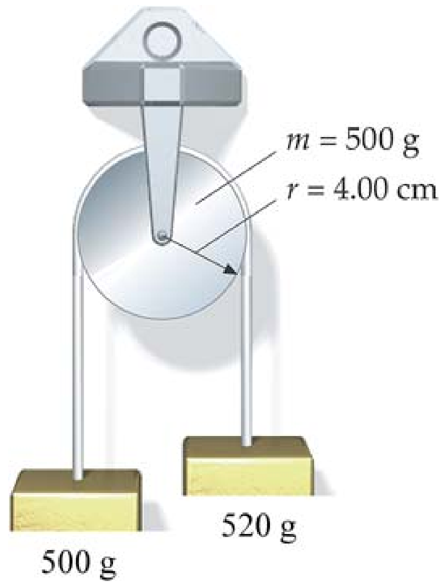
1a) (5 points) What is the tension in the cord holding the beam at an angle to the wall?

Solution
The key to this question is to place a rotation axis at the pivot, because then we do not need to deal with the unknown forces acting into and along the wall. Using $\tau = rF \sin\theta$, we can write out the torque balance from the pivot as:
(5 m)T = (15 m)(360 kg)(9.8 m/s^2) sin(60°), or T = 9166 N.

1b) (5 points) What is the magnitude of the force acting on the pivot along the wall (i.e., in the y-direction) and is it acting upwards or downwards?

Solution
The tension in the cord is acting upwards with a force of (9166 N) cos(30°) = 7938 N. The mass is acting downwards with (360)(9.8) = 3528 N of force. Therefore the pivot must have a force acting downwards on it of 7938 – 3528 = 4410 N. Notice that trying to calculate this y-force by placing a rotation axis along the rod and calculating the torques is complicated, because there are *two* torques acting on the pivot: one along the wall and one into the wall (x-axis force).

2) (10 points) Consider the Atwood's Machine shown at right. If the weights are motionless, then I release them, at what rate will the 520 g mass accelerate downwards?



Solution

Let us label the tension in the cord above the 500 g mass as T_1. Likewise, the tension above the 520 g mass will be T_2. Then we can write down three free-body equations, one for each mass and one for the wheel. We have:

$m_1 g - T_1 = m_1 a$
$-m_2 g + T_2 = m_2 a$
$rT_1 - rT_2 = I\alpha$

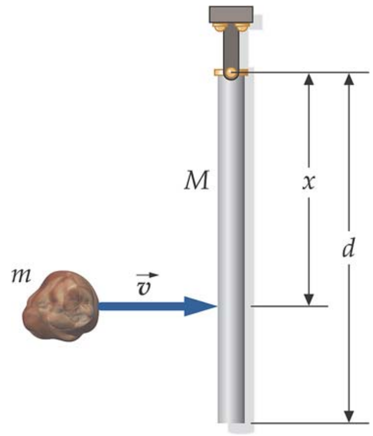
where $I = \frac{1}{2} mr^2$ and $a = \alpha r$.

Carrying out the algebra (substitute equations 1 and 2 into equation 3), we have:
$r(m_1 g - m_1 a - m_2 a - m_2 g) = \frac{1}{2} mr^2 (a/r)$, or $(m_1 - m_2)g = (\frac{1}{2} m + m_1 + m_2)a$.

$a = -(0.02)(9.8) / (0.25 + 0.5 + 0.52) = -0.154 \text{ m/s}^2$

Note that you cannot set the tension for each cord as simply equal to the weight of the mass below it. This is because the system is *moving* (i.e., accelerating), so the tension must be different from mg in order to move the masses.

3) A thin rod of length d = 1.00 m and mass M = 1 kg is hanging vertically from a frictionless pivot. Then, a mass of sticky clay with m = 400 g is shot onto the rod at x = 60 cm from the pivot at a velocity of v = 1.5 m/s. It sticks to the rod.



3a) (6 points) At what angular speed ω will the rod be turning instantly after the clay hits it?

Solution

The angular momentum of the clay about the pivot is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ = mvr = (0.4)(1.5)(0.6) = 0.36 kg m^2/s. This will be conserved. The angular momentum of the system afterwards is given by L = Iω, where I is the moment of inertia of the rod plus the clay. This moment of inertia is given by:

$I = {}^1/_3\, Md^2 + mx^2 = (0.333)(1)(1^2) + (0.4)(0.6^2) = 0.477$ kg m^2. Thus the initial angular velocity of the system will be ω = 0.36 / 0.477 = 0.755 rad/s.

3b) (8 points) What angle from the vertical (i.e., as measured from its original position) will the rod reach before it stops rising? Hint – what does the CM of the system do?

Solution

The "afterwards" rotational kinetic energy of the rod-clay system is given by $E = ½ I\omega^2 = (0.5)(0.477)(0.755)^2 = 0.136$ J. (Notice that we cannot use the kinetic energy of the clay *before* it hits the rod because this was not an elastic collision.) The center of mass of the rod-clay system (as measured from the pivot) is at y_{CM} = [(50 cm)(1 kg) + (60 cm)(0.4 kg)] / 1.4 kg = 52.86 cm, straight down, at the time of collision. The CM will rise until the gravitational potential gained equals the kinetic energy lost: in other words, we have 0.136 J = mgh, or h = 0.136 / (1.4)(9.8) = 0.99 cm. We therefore have cosθ = 51.87 / 52.86, or θ = 11°.

4) You are aboard a rocket in deep space which has a total mass of 2.8×10^5 kg. Half of this mass is fuel. You ignite the fuel and burn it all in exactly five minutes. The burning fuel has a speed relative to the rocket of 4 km/s.

4a) (4 points) When you first ignite the fuel, what is the acceleration of the rocket?

Solution

The thrust provided by the burning fuel is $F = (dm/dt)v = [(1.4 \times 10^5)/300](4000) = 1.867 \times 10^6$ N. The rocket's initial acceleration is thus $a = F/m = 1.867 \times 10^6 / 2.8 \times 10^5 = 6.67\ m/s^2$.

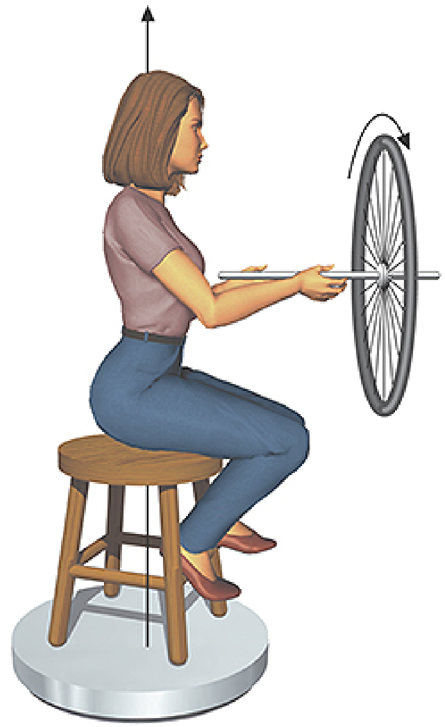
4b) (4 points) What is the final speed of the rocket?

Solution

The simple rocket equation is $v_F = v_o \ln(1 + M_{fuel} / M_{rocket})$, so we have:

$v_F = (4000) \ln(1 + 1) = 2773$ m/s.

5) A young lady is sitting on a frictionless turntable with the CM of her and her chair directly above the turntable's rotation axis. She is holding a bicycle wheel horizontally which has a radius of 31 cm and a mass of 3 kg. You may assume that the wheel's spokes are massless, and you may also ignore the mass of the wheel's rod and the lady's arms.



5a) (4 points) The CM of the bicycle wheel is 60 cm from the turntable's axis of rotation, and the bicycle wheel is rotating at $\omega = 25$ rad/s. What is the magnitude of the torque acting on the bicycle wheel?

Solution

The problem gives you a blizzard of information in an attempt to confuse you, but the torque acting on the bicycle wheel can only come from gravity acting on the CM of bicycle wheel. This is $\tau = r \times F = rmg = (0.6)(3)(9.8) =$ 17.6 N-m.

5b) (4 points) If we assume that the lady + chair + turntable have a small moment of inertia as compared to the outstretched bicycle wheel, calculate about how fast the lady is spinning.

Solution

If the lady + chair + turntable have no moment of inertia, then we just have a simple gyroscope. In that case $\Omega = Rg / fr^2 \omega = (0.6)(9.8) / (0.31^2)(25) =$ 2.45 rad/s, where we have taken f = 1 for a hoop.