Phyx 135-3-02 Final Solutions
Winter 2015

1) (10 points) A muon is an unstable particle with a half-life of 1.52×10^{-6} s. An alien spaceship with an muon-rocket propulsion system is passing the Earth at 0.5 *c* while firing a muon beam out the back of the spaceship at a speed of 0.9 *c* relative to the spaceship. What is the half-life of the muons from your point of view on Earth?

Solution

First we must find the speed of the muons as viewed from Earth. Taking the forward direction of the spaceship to be positive, we have: $v_1 = 0.5c$ and $v_2 = -0.9c$. This yields: $v = (v_1 + v_2) / (1 + v_1 v_2/c^2) = (0.5c - 0.9c) / (1 - 0.5 \times 0.9) = 0.7273c$. The time dilation effect then makes the half-life of the muons as seen from Earth: $\Delta t = \Delta t_0 / (1 - v^2/c^2)^{1/2} = (1.52 \times 10^{-6}\text{ s}) / (1 - 0.7273^2)^{1/2} = 2.21 \times 10^{-6}\text{ s}$.

2) (10 points) You are observing distant galaxies with the Hubble Space Telescope when you see the green color of a spectral line of the rare element Hypotheticalium. Your measurements reveal that this spectral line has a wavelength of 520 nm, with an error of $\pm$ 1 nm. Since your instruments are perfect, you know this means the error is due *solely* to basic quantum mechanics. Use this information to estimate how long excited electrons remain in that quantum level of Hypotheticalium before decaying.

Solution

We can use $\Delta E \, \Delta t = h / 2\pi$ to estimate the lifetime of the electrons. First however, we must convert the error in wavelength into an error in energy. There are at least three ways to do this.

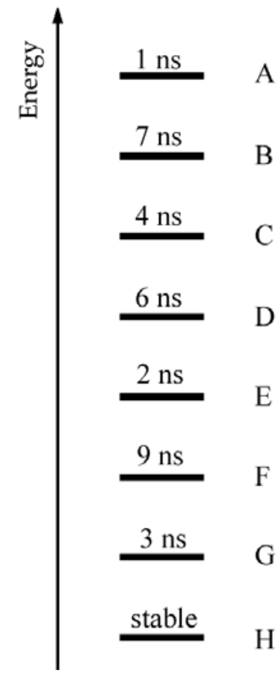
One way is to compute the energy for the "lower" side of the line: $E = hc / \lambda =$ $(6.63 \times 10^{-34})(3 \times 10^{8}) / (519 \times 10^{-9}) = 3.832 \times 10^{-19}$ J, and then do the same for the "higher" side of the line: $1.989 \times 10^{-16} / 521 = 3.818 \times 10^{-19}$ J. Subtracting these two yields $\Delta E = 0.0146 \times 10^{-19}$ J.

Another way is to note that the percentage wavelength error [1 nm – (–1 nm)] / 520 nm = 2/520 = 0.00385, and the percentage error in the energy must be the same. So, if $E = hc / \lambda =$ $(6.63 \times 10^{-34})(3 \times 10^{8}) / (520 \times 10^{-9}) = 3.825 \times 10^{-19}$, then $\Delta E = (0.00385)(3.825 \times 10^{-19}) =$ 0.0146×10^{-19} J, as before.

Last (and probably least), you can compute ΔE directly from $\Delta\lambda$ if you remember your basic calculus: $\Delta E = \Delta[hc / \lambda] = -hc \, \Delta\lambda / \lambda^2 = (6.63 \times 10^{-34})(3 \times 10^{8})(2 \times 10^{-9}) / (520 \times 10^{-9})^2 = 0.0145 \times 10^{-19}$ J, again as before.

However you compute ΔE, we have: $\Delta t = (6.63 \times 10^{-34})/(2\pi)(0.0146 \times 10^{-19}) = 7.5 \times 10^{-14}$ s.

3) At right is a schematic of the energy levels of the very rare element Hypotheticalium. The bottom level is the ground level; the energy of the levels increases as you go up. The time shown for each level is the half-life (in nanoseconds) of any electron excited into that level.



a) **(3 points)** If I excite a large number (~ 10^{20}) of Hypotheticalium atoms with an electric discharge, what is the maximum number of spectral lines (of any wavelength) the gas may give off?

Solution

Level A can decay down to 7 other levels, B can decay to 6, and so on.
7 + 6 + 5 + 4 + 3 + 2 + 1 = 28 lines.

b) **(2 points)** Which energy level is going to give rise to the faintest (fewest photons per second) spectral line?

Solution

Level F has the longest half-life, hence it will have the fewest transitions on average, hence it will give rise to the faintest spectral line.

c) **(3 points)** If the shortest wavelength radiated by the Hypotheticalium is 150 nm, what is the energy difference between level A and level H?

Solution

Using E(eV) = 1240 / λ(nm), we have E = 1240/150 = 8.27 eV

4) The Bohm-Aharonov Effect is an exotic quantum phenomena which results when an electron is diffracted around a magnetic field. When I was in graduate school, another student asked my advice about measuring this effect using a clunky old magnet that he'd found in a storage closet.

a) (5 points) His magnet was 5 cm in diameter. Estimate the maximum kinetic energy (in joules) that an electron can have if it is to experience any significant degree of diffraction around an object of this size. State *very briefly* how you are making your estimate.

Solution

To show significant diffraction, the electron's wavelength should be roughly as large as the cylinder. We estimate λ to be 5 cm. At this point a number of students used $E = hc/\lambda$ to relate λ to E, but this formula is only good for photons (i.e., zero rest mass). For electrons we have $\lambda = h/p = h/mv$, so $v = h/m\lambda$ and thus $E = ½ mv^2 = h^2 / 2\lambda^2 m = (6.63 \times 10^{-34})^2 / (2)(0.05)^2(9.11 \times 10^{-31}) = 9.65 \times 10^{-35}$ J

b) (3 points) Assume that the inside of your vacuum chamber for measuring the Bohm-Aharonov Effect is sealed off from all light, radio waves, or other external radiation. However, the chamber is at room temperature, or 290 K. What would be the most predominant energy of the remaining photons?

Solution

The peak in the blackbody radiation of an object at 290 K occurs at $\lambda = 2.90 \times 10^{-3} / T = 2.90 \times 10^{-3}/290 = 10^{-5}$ m. The photon energy is $E = hc/\lambda = (6.63 \times 10^{-34})(3 \times 10^{8})/(10^{-5}) = 1.99 \times 10^{-20}$ J.

c) (2 points) Using your answers to Parts a and b, give one specific reason why I all but fell on the floor laughing after the student left my lab.

Solutions

The ratio between the electron energy and the energy of the infrared photons is an incredible $(1.99 \times 10^{-20})/(9.65 \times 10^{-35}) = 2.06 \times 10^{14}$! Interaction with even one photon would blast the electron to an energy 100 billion times too hot to diffract.

Also, if you calculate the velocity of the electron using $v = h/ml = (6.63 \times 10^{-34})/ (9.11 \times 10^{-31})(0.05) = 0.0146$ m/s, you can see that it would take the electron $0.0146^{-1} = 68.5$ seconds to travel one meter. But in that time, gravity would cause the electron to fall $d = ½ gt^2 = 0.5(9.8)(68.5)^2 = 22{,}987$ meters, or about 14 miles! Good luck hitting the target.

5) (10 points) Natural uranium today consists of 99.7% $^{238}U_{92}$ and 0.3% $^{235}U_{92}$. Given half-lives of 4.468×10^9 years for U-238 and 7.038×10^8 years for U-235, what would have been the relative percentages of U-238 and U-235 in natural uranium when the Earth was formed 4.5 billion years ago?

Solution

$N(t) = N_o \exp(-t \ln2/\tau)$ means $N_o = N_{now} \exp(t \ln2/\tau)$. Using billions of years as our time unit, we see that every 99.7 atoms of U-238 now existing corresponds to $N_o = 99.7 \exp(4.5 \ln2 / 4.468) = 200.4$ atoms in 4.5 billion BC. Likewise for U-235, $N_o = 0.3 \exp(4.5 \ln2 / 0.7038) = 25.2$ atoms then.

The abundance of U-235 was thus 25.2/(200.4 + 25.2) = 11.2% in 4.5 billion BC. The U-238 made up 100 – 11.2 = 88.8%

6) (10 points) A catfish is floating 2.00 m below the surface of a smooth lake. What is the diameter of the circle on the surface through which the catfish can see the world outside the water? (Assume that $n = {}^4/_3$ for water.)

Solution

Light barely grazing the water at 90° will be refracted towards the fish at a critical angle given by:
$n_1 \sin\theta_c = n_2 \sin(90°)$, where $n_1 = {}^4/_3$ and $n_2 = 1$. We have $\theta_c = \sin^{-1}({}^3/_4) = 48.59°$

It might have been useful for you to draw a little picture at this point, in order to see more clearly that $r / h = \tan\theta_c$, where r = the radius of the circle of light on top of the water, and h = the depth of the catfish in the water. In any case, we have $D = 2r = 2h \tan\theta_c = (2)(2.00 \text{ m})\tan(48.59°) =$ 4.54 m.

7) (6 points) Carbon-14 undergoes β-decay into nitrogen-14. What is the maximum kinetic energy that the antineutrino produced in this reaction can carry away? (Please convert your answer to keV.)

Some atomic masses, and other numbers

^{14}C = 14.003241989 u

^{13}C = 13.003354838 u

e = 5.4858×10^{-4} u

u = $1.660538782 \times 10^{-27}$ kg

1 eV = 1.602×10^{-19} J

^{14}N = 14.0030740048 u

^{15}N = 15.0001088982 u

n = 1.00866491597 u

c = 2.998×10^{8} m/s

Solution

We will neglect the rest mass of the antineutrino. The relevant reaction then becomes: $^{14}C \rightarrow {}^{14}N + e$. The difference in mass between the two sides will be the energy released, which is also the maximum kinetic energy that the antineutrino can carry away. We remember that *atomic* masses include the masses of the relevant number of electrons, or in other words,

Atomic mass of carbon = carbon nucleus + 6 electrons
Atomic mass of nitrogen = nitrogen nucleus + 7 electrons

So, when the carbon nucleus becomes a nitrogen nucleus + electron (β-decay), the difference between the before-and-after masses is just the carbon atomic mass minus the nitrogen atomic mass.

$\Delta E = (14.003241989 - 14.0030740048)(1.660538782 \times 10^{-27})(2.998 \times 10^{8})^2 = 2.50709 \times 10^{-14}$ J,
or 156.5 keV

Multiple Choice Questions (2 points each)

__**E**__ **8)** When I turn on my computer monitor, and the screen lights up, the mass of the monitor:

A) begins to radiate away
B) depends on what software I am using
C) decreases
D) stays the same
E) increases
F) is partly converted to electricity

Comment: Adding energy increases the mass of the monitor, by $E = mc^2$.

__**B**__ **9)** I am shining light of a single wavelength on the surface of a metal, but no photoelectrons are being emitted. To rectify this problem, I should:

A) use light of a longer wavelength
B) use light of a shorter wavelength
C) use the same light but increase the intensity
D) use the same light but decrease its intensity
E) use a laser with the same wavelength
F) heat the metal

Comment: You need more energy per photon, and $E = hc/\lambda$.

__**F**__ **10)** Which of the following is true for the photons in a laser beam?

A) They have the same frequency
B) They have a short half-life
C) They can only be red or green
D) They have the same polarization
E) They have the same phase
F) A, D, and E
G) A and B
H) C and D

__**F**__ **11)** How many values of the magnetic quantum number "m" are possible for an electron if it is in an $l = 8$ state?

A) 9 B) 15 C) 5 D) 8 E) 3 F) 17

Comment: The total number of "m" states is always $2l + 1$.

__**C**__ **12)** β-decay occurs in an unstable nucleus when:

A) a proton is converted to an electron by the strong force
B) a proton is converted to a neutron by the strong force
C) a neutron is converted to a proton by the weak force
D) a neutron is converted to an alpha particle by the weak force
E) a neutron is converted to a beta particle by the weak force
F) an alpha particle escapes from the nucleus

__**E**__ **13)** $^{235}U_{92}$ is radioactive and decays to $^{227}Th_{90}$, but not with one reaction. It decays using a series of reactions. In this series, the particles ejected must consist of:

A) one α-particle and three β-particles
B) three α-particles and one β-particle
C) one α-particle and four β-particles
D) two α-particles and one β-particle
E) two α-particles and two β-particles
F) one α-particle and two β-particles

Comment: You need two α-decays to reduce the weight of the U-235 to 227. But the α-decays reduce the atomic number to 88, so you need two β-decays to lift it back to 90.

__**A**__ **14)** The wavelength of a beam of light passing through a liquid is 360 nm, but it changes to 469 nm when the beam of light leaves the liquid and enters a vacuum. What is the index of refraction of the liquid?

A) 1.30	B) 1.05	C) 1.50	D) 1.70
E) 1.90	F) 1.10	G) 1.43	

Comment: $\lambda = \lambda_{vac}/n$, so $n = \lambda_{vac}/\lambda = 469/360 = 1.30$

__**C**__ **15)** Which of the following nuclear reactions is impossible? (e = electron, n = neutron)

A) $n + {}^{235}U_{92} \rightarrow {}^{90}Sr_{38} + {}^{136}Xe_{54} + 10n$

B) ${}^{238}U_{92} + n \rightarrow {}^{239}Pu_{94} + 2e + 2$ anti-neutrinos

C) ${}^{228}Th_{90} \rightarrow {}^{228}Ac_{89} + e$ + anti-neutrino

D) three ${}^{4}He_{2} \rightarrow {}^{12}C_{6}$

E) ${}^{16}O_{8} + {}^{4}He_{2} \rightarrow {}^{20}Ne_{10}$

F) ${}^{3}H_{1} \rightarrow {}^{3}He_{2} + e$ + anti-neutrino

Comment: β-decay raises the atomic number, it does not lower it.

__**B**__ **16)** The three rare-earth elements gadolinium, terbium, and dysprosium are elements 64, 65, and 66, respectively. Which one of the following statements is true?

A) Gadolinium is the rarest of the three rare earths.

B) Terbium is the rarest of the three rare earths.

C) Dysprosium is the rarest of the three rare earths.

D) Gadolinium and terbium are equally rare.

E) Terbium and dysprosium are equally rare.

F) Gadloinium and dysprosium are equally rare.

Comment: As discussed in class, due to the details of stellar nucleo-synthesis, the odd-numbered elements above carbon are *always* more rare than the even-numbered elements to either side.