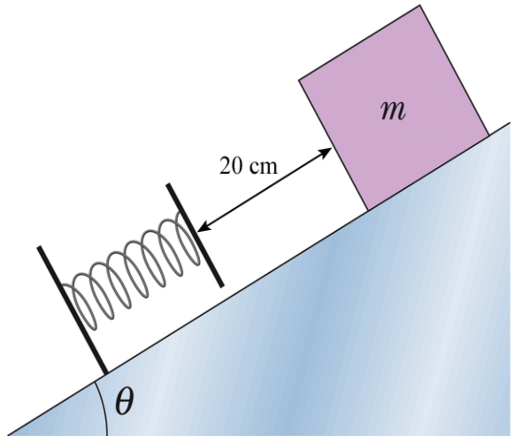
Phyx 135-1 Midterm I, Spring 2015 **Name** ________________________________

1) (12 points) A block of mass m = 2.5 kg is setting on a ramp inclined at $\theta = 37°$. There is a coefficient of kinetic friction $\mu_k = 0.15$ between the ramp and the block. The block is 20 cm (*along* the ramp) from a spring with a spring constant of k = 30 N/m. If I release the block and let it slide freely until it just stops, by what distance x (measured along the ramp) will it have compressed the spring?



Solution

Conservation of energy tells us that the energy released by the block as it falls must go somewhere. In this case, it will go into the potential energy of the spring and into frictional heat. If we let x equal the amount the spring is compressed, then the block will descend a total distance of 0.2 m + x along the ramp. This corresponds to a change in height of $h = (0.2 + x)\sin(37°)$, so the block will lose $E = mgh = mg(0.2 + x)\sin(37°)$ of energy.

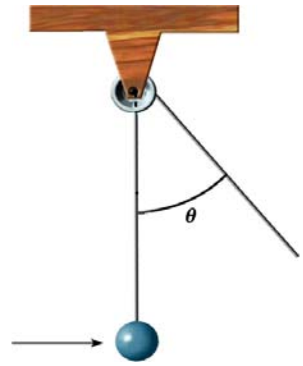
Meanwhile, as it slides, the block will generate frictional heat given by $W = Fd = \mu_k Nd = \mu_k\, mg \cos(37°)\, d = \mu_k\, mg \cos(37°)(0.2 + x)$. Finally, whatever energy is left over will be absorbed by the spring, which has $E = ½\, kx^2$.

Putting all this together gives us the equation:
$mg(0.2 + x)\sin(37°) = \mu_k\, mg \cos(37°)(0.2 + x) + ½\, kx^2$

Inserting numbers:
$(2.5)(9.8)(0.2 + x)(0.6018) = (0.15)(2.5)(9.8)(0.7986)(0.2 + x) + 15x^2$, or
$11.81(0.2 + x) = 15x^2$, or $15x^2 – 11.81x – 2.36 = 0$.

Using the quadratic equation: $2ax = –b \pm (b^2 – 4ac)^{1/2}$, or
$30x = 11.81 \pm (139.48 + 141.6)^{1/2} = 11.81 \pm 16.77$. We reject the negative sign as unphysical, and thus have $x = (11.81 + 16.77)/30 =$ 0.953 m.

2) (10 points) Suppose a ball of mass m = 5 kg is hanging by a string from a frictionless pivot. The length of the string is 2 meters. Then, a bullet of mass m = 35 g travelling at 600 m/s is shot horizontally into the ball (i.e., the bullet remains stuck inside the ball). To what angle θ will the ball rise?

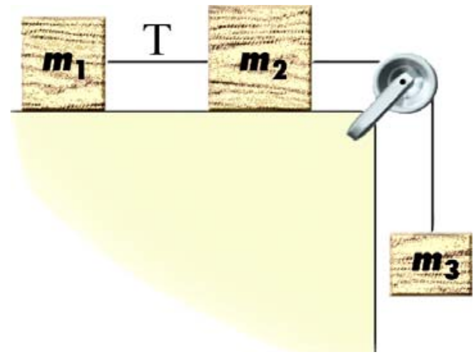


Solution

There are two parts to this problem. The first requires that we use conservation of momentum. The momentum of the bullet is p = mv = (0.035)(600) = 21 kg m/s. After it hits the ball, the velocity of both of them will be given by: 21 = mv = (5 + 0.035)v, or v = 4.17 m/s.

For the second part, we realize that the ball will rise until it runs out of kinetic energy. In other words, we have $mgh = \frac{1}{2} mv^2$. Or, $h = v^2/2g = (4.17)^2/2(9.8) = 0.887$ m. We thus have: $\cos\theta = (2 - 0.887)/2 = 0.5565$, or $\theta = 56°$.

3) (10 points) Three masses of m_1 = 2 kg, m_2 = 3 kg, and m_3 = 1 kg are arranged at shown at right. There is a coefficient of kinetic friction $\mu_k = 0.25$ between mass m_2 and the table; the rest of the system is frictionless. If I allow m_3 to fall freely, what will be the tension T in the cord between masses m_1 and m_2?

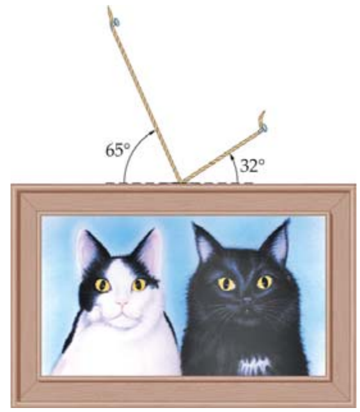


Solution

We see that there are only two forces operating on the three masses taken together: m_3g pulling cw, and $F = \mu_k m_2 g$ resisting ccw. Thus the acceleration of the system is given by F = ma, or $m_3g - \mu_k m_2 g = (m_1 + m_2 + m_3)a$. Inserting numbers:
(1 kg)(9.8 m/s^2) – (0.25)(3 kg)(9.8 m/s^2) = (2 kg + 3 kg + 1 kg)a, or a = 0.4083 m/s^2.

The tension T between masses m_1 and m_2 only has to accelerate m_1 to the right, so we have:
$T = F = m_1a$ = (2 kg)(0.408 m/s^2) = 0.817 N.

4) (10 points) You tack a photo to the wall by using two pieces of string that are rather sloppily attached to the photo at angles of 65° and 32°, respectively, for the left and right strings. If the photo has a mass of 5 kg, what are the tensions in the two strings?



Solution

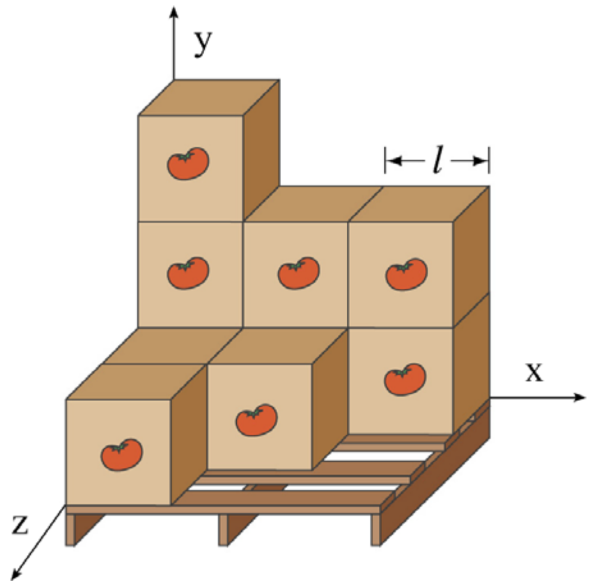
We know that the x-axis forces must add to zero (because the photo is not moving), whereas the y-axis forces must add to F = mg. We therefore have:

$T_L \cos(65°) = T_R \cos(32°)$ (x-axis forces)

$T_L \sin(65°) + T_R \sin(32°) = (5)(9.8)$ (y-axis forces)

The first equation gives us $T_L = 2.007\ T_R$, so substitution into the second equation gives: $2.007\ T_R(0.9063) + T_R(0.5299) = 49$, or $T_R = 20.86$ N. Then $T_L = 41.87$ N.

5) (8 points) If the coordinate origin of the boxes shown at right is chosen to be at the far back left corner of the pallet, then where is the center of mass of the boxes? (There are 10 boxes total on the pallet.) Take l = 1 meter.



Solution

The x-coordinate of the CM will be:
[5(0.5) + 3(1.5) + 2(2.5)] / 10 = 1.2 m.

The y-coordinate of the CM will be:
[6(0.5) + 3(1.5) + 1(2.5)] / 10 = 1 m.

The z-coordinate of the CM will be:
[7(0.5) + 2(1.5) + 1(2.5)] / 10 = 0.9 m.