

1) (10 points) An old-style vinyl LP record has a mass of 200 g and a radius of 15.25 cm. If the turntable provides a torque of 0.006 N-m to the record, how long will it take the record to reach its final speed of $33 \frac{1}{3}$ revolutions per minute?

Solution

We can use $\tau = I\alpha$ to find the angular acceleration. In this case, $I = \frac{1}{2}MR^2 = \frac{1}{2}(0.2)(0.1525)^2 = 2.326 \times 10^{-3} \text{ kg m}^2$. Thus $\alpha = 0.006 / (2.326 \times 10^{-3}) = 2.58 \text{ radians/s}^2$. A rotation speed of 33.33 rpm means $f = (33.33 / 60) \text{ Hz}$, or $\omega = 2\pi f = 2\pi(33.33)/60 = 3.49 \text{ rad/s}$. Finally, using $\omega = \alpha t$ gives us $t = 3.49 / 2.58 = 1.35 \text{ s}$.

2) (10 points) Suppose the LP record in Problem 1 is spinning away at its final speed of $33 \frac{1}{3}$ rpm when the record changer accidentally drops a smaller 45 rpm record (radius = 8.9 cm, mass = 68 g) onto the turntable, directly on top of the LP. The turntable's motor instantly shuts off and just lets the two records spin. At what rate (in rpm) will they be rotating after the 45 rpm record stops sliding across the LP?

Solution

With no torque being supplied, the records will conserve angular momentum, $L = I\omega$. In other words, we must have $I_1\omega_1 = I_2\omega_2$, or a simple ratio. The initial moment of inertia (from Prob 1) is $2.326 \times 10^{-3} \text{ kg m}^2$, and afterwards $I = 2.326 \times 10^{-3} + \frac{1}{2}(0.068)(0.089)^2 = 2.595 \times 10^{-3} \text{ kg m}^2$. The ratio of the two I's is the inverse ratio of the rotation speeds, so we can skip some algebra and calculate the new rate to be $(33.33)(2.326 \times 10^{-3}) / (2.595 \times 10^{-3}) = 29.9 \text{ rpm}$.