

First Midterm Solutions, Phyx 135-2, Lecture 01, Fall 2013

1) A charge of $+3.0 \mu\text{C}$ is located at $x_1 = 3.5 \text{ cm}$ and $y_1 = 0.50 \text{ cm}$. A second charge of $-4.0 \mu\text{C}$ is located at $x_2 = -2.0 \text{ cm}$ and $y_2 = 1.5 \text{ cm}$.

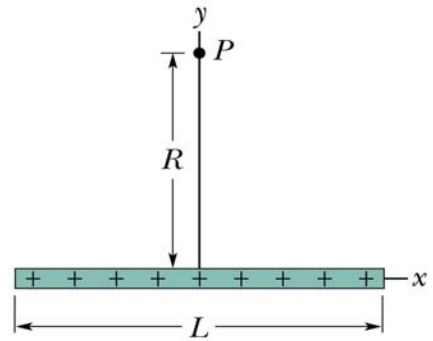
a) What is the magnitude of the electrostatic force operating between them?

b) If the positive x-axis defines $\theta = 0$, at what angle is the force operating on charge 1?

Solution: The distance r between the two charges is $[(3.5 + 2.0)^2 + (0.5 - 1.5)^2]^{1/2} = 5.59 \text{ cm}$. The force between them is $F = kq_1q_2/r^2 = (8.99 \times 10^9)(3 \times 10^{-6})(-4 \times 10^{-6})/(0.0559)^2 = -34.5 \text{ N}$. The minus sign indicates that the force is attractive.

The force on charge 1 points from $(3.5, 0.5)$ to $(-2, 1.5)$, thus the force vector is pointing in a direction that is “5.5 cm to the left and one cm up”. The arctangent of $1 / 5.5$ is 10.3° , in the second quadrant, thus the angle in question is $(90^\circ - 10.3^\circ) + 90^\circ = 169.7^\circ$

2) Suppose you have a uniform, nonconducting rod of length L which contains a total charge of q distributed along it. Write down a definite integral that would give you the electric field at a point P which is midway along the rod and a distance R from it. (You do not have to solve the integral, just write it down.) For partial credit, it would be wise to explain what you are doing.



Solution: We start with the basic formula $dE = (k dq / r^2) \mathbf{r}$.

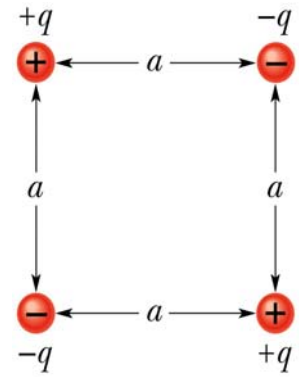
We need to convert everything into a single variable that we can then integrate. The most reasonable choice is to use “ x ”, so that we can integrate along the rod from $+L/2$ to $-L/2$.

dq can be related to a very short distance along the rod, dx , by using the ratio $dq = (q/L) dx$. (In other words, $dq = \lambda dx$, where $\lambda = q/L$.) Next, we realize that r is the distance from P to the element dx , which in this case we can write as $r^2 = R^2 + x^2$. As for \mathbf{r} , we see by symmetry that the electric field at P will point directly upward along the y -axis, and also that the component of the field in the y -direction will be given by R / r . Substituting everything into our basic formula gives us: $dE = [k (q/L) dx / (R^2 + x^2)] (R / r) = kqR dx / L(R^2 + x^2)^{3/2}$. We can then integrate this from $+L/2$ to $-L/2$, or from 0 to $L/2$ after multiplying by 2 .

3) Two infinite, nonconducting sheets of charge are parallel to each other and located a distance $d = 40$ cm apart. They contain equal but oppositely signed amounts of charge. If the potential difference between the sheets is 200 volts, how much charge is contained (on either sheet) within an circle of radius = 2 meters?

Solution: We have $E = 200 \text{ volts} / 0.4 \text{ m} = 500 \text{ volts/m}$. We know $E = \sigma / 2\epsilon_0$ for a *single* infinite sheet of nonconducting charge, but since we have two sheets of opposite charge parallel to each other, the electric field from only one of them is 250 volts/m. Thus we have $\sigma = 2E\epsilon_0 = (2)(250)(8.85 \times 10^{-12}) = 4.425 \times 10^{-9} \text{ C/m}^2$. The amount of charge on the circle will be $\sigma \pi r^2 = (4.425 \times 10^{-9})(3.14159)(2)^2 = 5.56 \times 10^{-8} \text{ C}$.

4) Suppose four charges of magnitude $q = e = 1.6 \times 10^{-19}$ C are arranged as shown at right. $a = 5$ nm. How many eV of energy would you need to pull the charges apart and place them at infinity?



Solution: We have $V = kq/r$ for a point charge, and $E = qV$. As discussed in class, we can remove any of the charges in any order and still get the same answer for the total electrostatic potential energy, so let us start at the top left and go clockwise. This charge has $-ke^2/a + ke^2/\sqrt{2}a - ke^2/a$ of energy. After removing it, the top right charge has $-ke^2/a + ke^2/\sqrt{2}a$ of energy. After removing those two, the bottom right charge has $-ke^2/a$ of energy. After removing those three, the bottom left charge has zero energy.

The sum of the potentials is $E = -ke^2/a + ke^2/\sqrt{2}a - ke^2/a - ke^2/a + ke^2/\sqrt{2}a - ke^2/a = -4ke^2/a + 2ke^2/\sqrt{2}a = (ke^2/a)(\sqrt{2} - 4)$, so we must add $(ke^2/a)(4 - \sqrt{2})$ of energy to pull them apart. The energy in eV is: $E/e = (ke/a)(4 - \sqrt{2}) = (2.586)(8.99 \times 10^9)(1.6 \times 10^{-19})/(5 \times 10^{-9}) = 0.744$ eV.