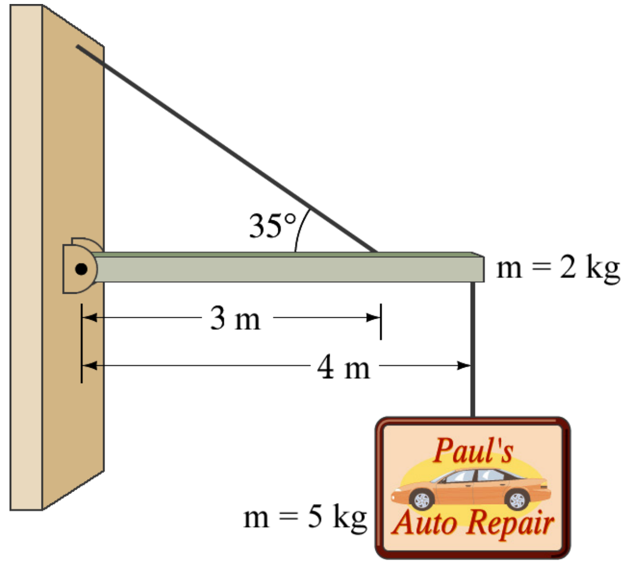
Suppose we have a beam with a mass of 2 kg that is resting against a wall. There used to be a bolt in the pivot joint, but some mischievous college students – obviously deranged by physics deprivation as a result of spending too little time on their physics homework problems – have removed it. Only friction with the wall is holding the beam in place. There is a cable and a sign attached to the beam, as shown in the figure. What is the tension in the cable? What minimum coefficient of friction do I need to keep the beam from moving?



Solution: All forces and all torques must sum to zero in a statics problem. In this case there is no point in looking at the x-forces from the wall, because the wall cannot be moved and it will simply counter any forces that are placed on it. The reaction forces have to be there, of course, according to Sir Isaac, but they provide us with zero information because you can't calculate them unless you already know the forces coming from the beam and the cable!

So, let's start instead by looking at the torque. The net torque in a static system must be zero no matter what axis of rotation is used. Since we have an unknown force acting at the pivot, let's put our axis of rotation there. This has the tremendous advantage that $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}_{PIVOT} = 0$ because $r = 0$ if the pivot is our axis.

Using the normal positive counter-clockwise sign convention, we can write down the three nonzero torques acting around the pivot:

	radius	**force**	**θ**	**rF sinθ**
Repair Sign	4 m	–(5 kg)g	90°	–(4)(5)(9.8)(1) = –196 N-m
CM of beam	2 m	–(2 kg)g	90°	–(2)(2)(9.8)(1) = –39.2 N-m
Cable tension	3 m	+T	35°	3T sin(35°) = +1.721 T

Setting the torque equal to zero: $-196 \text{ N-m} - 39.2 \text{ N-m} + 1.721\ T = 0$, or $T = 136.7$ N.

Let us now look at the y-force acting on the pivot. In the y-direction, the weight of the sign and the beam is $-(5 \text{ kg} + 2\text{kg})(9.8 \text{ m/s}^2) = -68.6$ N. The y-component of the tension in the cable is $(136.7 \text{ N})(\sin 35°) = 78.4$ N. Thus we have an imbalance of $78.4 - 68.6 = 9.8$ N, which the pivot must provide.

Wait a minute! The cable's y-force is actually *more* than the weight of the system! What? Who? The pivot is pushing downward! Have we proven that the pivot must hold the cable down, lest the entire system break loose and would float off into the sky?!

Well, not exactly. Let's look at the torque again, but this time let's put our rotation axis at the point where the cable connects to the beam ($x = 3$m). If we let F_Y = the y-force being supplied by the pivot, then we have $F_Y(3 \text{ m}) + (2 \text{ kg})g(1 \text{ m}) - (5 \text{ kg})g(1 \text{ m}) = 0$, or $F_Y = (3 \text{ kg})g / 3 = 9.8$ N.

Aha! Now everything is clear. The Repair Sign is trying to rotate the beam clockwise (negative torque) around the point where the cable is connected to the beam. The pivot is providing 9.8 N of counter-clockwise force (positive torque) so that the beam will not rotate. But a y-force is a y-force no matter where it comes from, so the cable must counter both it *and* gravity, thus the cable is providing more positive y-force than the total weight of the system. (Don't you love statics?) If we greased the pivot and negated its friction, the beam would not float away – rather, the beam would rotate clockwise around the 3-meter point, then its CM would swing to the left and whack the wall.

Counter-intuitive "extra" forces like the pivot friction are not as outlandish as they might seem. Ask yourself, which is easier to do: hold your physics textbook over your head, or hold it at arm's length? $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ is just another way of saying that it is easier to lift a weight from directly below than it is to pull it at a 15° angle from 20 feet away. Systems with long bars/beams/cables in them often generate tensions that far exceed the actual weight of the system, which in turn means that you can end up with counter-intuitive balancing forces in places you don't expect.

Let us move on to the final answer. We wish to know the minimum coefficient of friction needed to hold the beam in place. We have $F_f = \mu N$, and in this case the friction is acting in the y-direction. We know that the minimum y-force needed to balance the beam is 9.8 N. The normal force acting to push the beam into the wall must equal the x-component of the tension in the cable, because there is no other x-force acting on the beam. This is $T_X = (136.7\text{ N}) \cos(35°) = 112\text{ N}$. The necessary coefficient of friction is a quite modest $\mu = 9.8 / 112 = 0.088$