

NUCLEAR DECAY OF NATURAL AIRBORN RADIATION THE RADON DECAY CHAIN

INTRODUCTION

Natural sources of radiation exist all around us. Cosmic rays bombard our atmosphere, creating the isotope C^{12} that is used in carbon dating procedures. Radioactive Thorium was present in lantern mantels until only recently. Bricks and cement products contain trace amounts of Thorium and Uranium as well as K^{40} , which is also found in human blood. It is thought that these radioactive sources originate from nuclear reactions occurring deep in the earth's interior. In this experiment a sample of airborne dust is collected and tested for radioactive contaminants. At the same time properties of radioactive decay will be studied. For the mathematical theory of radioactive decay please read the Appendix, but here we first go through the lab steps.

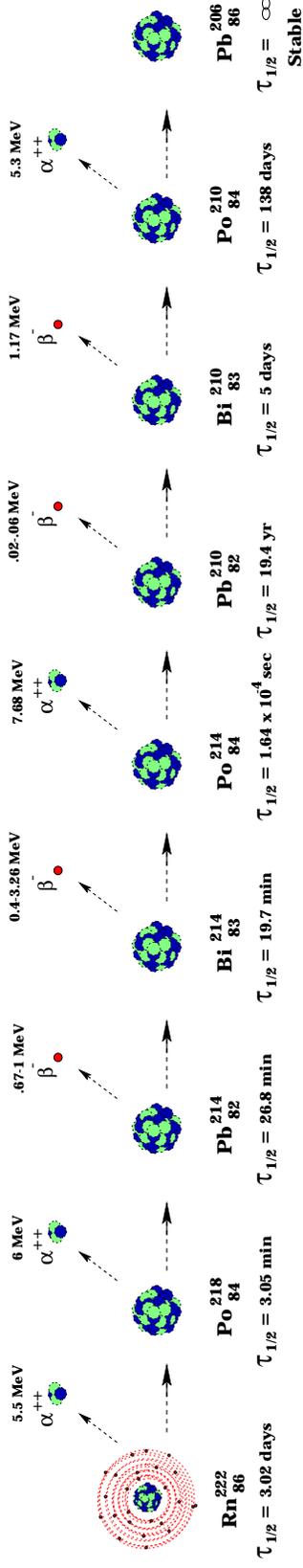
EXPERIMENT

You will be testing the decay of radioactive substances found in the air around us. Two of the daughter series of heavier radioactive nuclei found in the earth are isotopes of the noble gas radon. Since these isotopes, Rn^{222} (Radon) and Rn^{220} (Thoron) are non-reactive gases they may diffuse out of the earth into the air where they are attracted to positively charged aerosol particles. These isotopes have the decay sequences shown in Figures 2. Both decay sequences are evidently quite complicated.

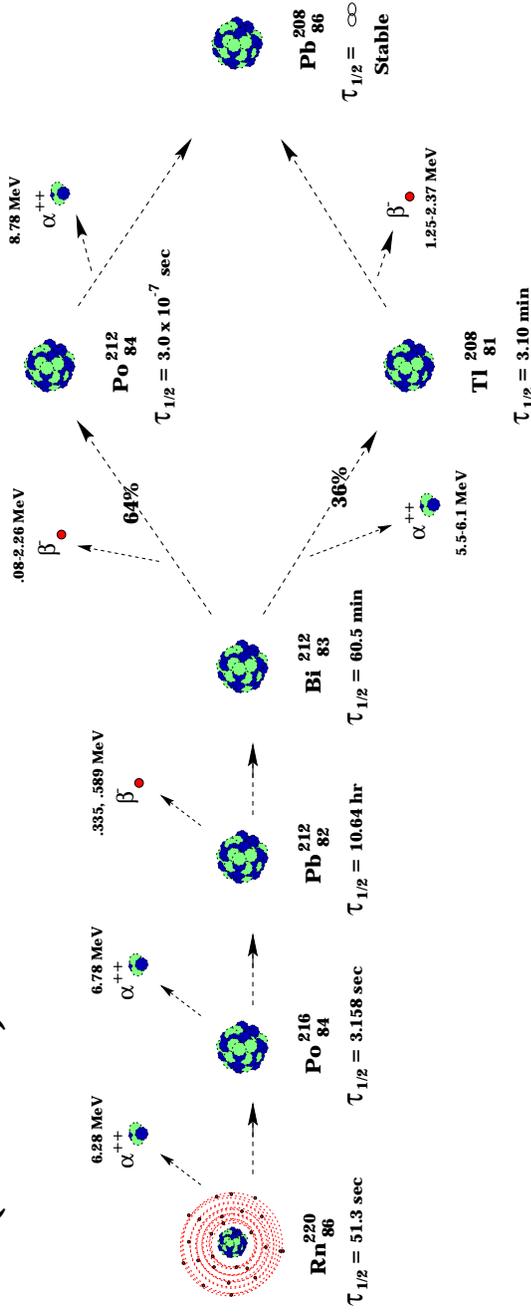
Since we will only be measuring the total radioactivity (decays counted per time interval) we are primarily interested in only the subsection(s) of the decay chain that show effects on the time periods used in the measurements. For example Rn^{222} undergoes a sequence of decays into Pb^{210} which has a half-life of 19.4 years. All subsequent daughter nuclei are stable or have a half-life of many days. As a result Pb^{210} and its daughter nuclei will contribute relatively few counts over the time periods during which you will be sampling. They may therefore be neglected when compared to counts resulting from the quickly decaying daughters of Rn^{222} higher up the decay chain. The section of the decay chain from Rn^{222} to Po^{210} may be further simplified by noting the quick decay of Po^{214} (1.64×10^{-4} sec). Any Bi^{214} nuclei that decay into Po^{214} will almost immediately decay further into Pb^{210} . Unless

Decay Chains of Radon Isotopes

Radon 222



Radon 220 (Thoron)



the radioactive event counting equipment being used has a very high time resolution the effects of the intermediate state Po^{214} will be negligible.

To collect the radioactive sample you will build an electric charge on both a Styrofoam cup and a balloon by rubbing them with a piece of fur that will deposit electrons. Positively charged aerosol and dust particles containing Rn^{222} and Rn^{220} will adhere to the cup and balloon when they are attracted by the charge. Inverting the cup on the lab table and hanging the balloon from the ceiling, rub them with the fur and then wait half an hour for the charge to dissipate and the collection of dust to cease. You might want to wait slightly longer to allow some short-lived products to decay away. At this point the balloon is popped and the bottom of the cup is sliced off so that they may be placed under the Geiger counter.

There are two programs that you may use to collect the data. One of them counts and stores the radioactive events detected during up to 160 consecutive one-minute time intervals. The other program counts events until it is stopped at which time it displays the length of the time interval and number of counts detected during it. At this point a new counting interval may be started. The time since the program was started is continuously displayed allowing the time at which each sample was taken also to be easily recorded.

Each program has its own advantages and disadvantages. The first program is fairly automatic but can only count for 160 minutes. The second program has less limitation on interval and total time duration but requires more human control. Since the activity of the balloon and cup is relatively low we run into one of the classic catch twenty-twos of statistical measurements. In order to obtain good statistics we must measure a lot of counts, however, since activity is low this may only be done by using long time intervals. This results in a compromise between the accuracy of the data samples vs. the amount of data points that may be taken over a particular time interval. The Part I of the Appendices in Lab 4, *Geiger Müller Counters and Radiation Event Counting Software*, gives some more details on using the Geiger counter and the computer software.

As shown in Figure 1, the daughter elements in the decay chains of Rn^{222} and Rn^{220} have various half-lives ranging from fractions of seconds to days and years. Our activity levels will not allow us to measure effects of decay constants corresponding to half-lives less than a couple minutes. And most students will probably prefer for the lab to last less than a week so half-lives of more than a day won't be detected.

In order to measure effects with shorter time scales take data with the first program and add the counts over sets of five or so consecutive one-minute intervals to get enough counts to give reasonable statistics for each data point as shown in Figure 2. Another alternative is to use a sliding window to smooth out the fluctuations in the data. For this technique, using a 5 bin wide window, the first five count samples are added then the second through sixth, third through eighth, and so on. The other program may be used to collect data for time periods on the order of an hour and thus measure effects of daughter nuclei with

5 minutes										1 minute									
← →										↔									
17	15	18	14	16	21	9	16	18	24	17	16	21	10	8	11	24	16	14	14
80					88					72					79				

Figure 2 Summing Counts of Consecutive Intervals to Improve Statistics

By adding consecutive sample points together larger number of counts are obtained and thus higher statistical accuracy. For example during the fifth original counting interval or bin 16 counts were detected for a standard deviation of about 4 giving an approximate accuracy of only $\approx \frac{\pm 4}{16} = \pm 25\%$. Adding the first five bins together gives 80 detected counts over 5 minutes with a standard deviation of ≈ 8.94 counts and accuracy of $\approx 11.2\%$. This is a little over twice as good accuracy as before. However, our ability to resolve changes in count rate with time is severely reduced since we are effectively sampling only one fifth as often, i.e. every 5 minutes instead of each minute.

5 minutes										1 minute									
← →										↔									
17	15	18	14	16	21	9	16	18	24	17	16	21	10	8	11	24	16	14	14
80 84 78					76 80 88 84 91					96 88 72 66 74					69 73 79				

Figure 3 Summing Counts Within a Sliding Window

The method of adding consecutive sets of samples together to get better statistics via higher count numbers described in Figure 2 severely reduces the sampling frequency. This effect may be ameliorated to some degree by using a sliding window. For this technique, if a window that is five bins wide is used, the first five samples are added, then the second through sixth, third through seventh, and so on. This helps to fill in some points between the ones obtained in Figure 2. These new sets of samples have higher accuracy than the initial samples since the numbers are higher. However, even though there is still a sample every minute as with the initial data, fluctuations in time get smoothed out somewhat.

longer half-lives on the counting statistics. You will also want to measure the average count rate of decays detected when the sample is not present as a control. This is referred to as the background count.

DATA ANALYSIS

MEASURING THE DECAY CONSTANTS

Subtract the background count from each counting measurement and plot the natural log of the data as a function of time. Choose a portion of the data for which the plot appears to be linear. Enter that data (actually the natural log of the data) into a linear regression program to obtain a half-life. Several sample MatLab scripts have been prepared that you may use to help you with this task. They may be downloaded from the C35 Matlab Scripts World Wide Web Page (<http://www.physics.nwu.edu/C35/PAGES/matscrip.htm>) along with some readme files.

Running the main script *decayfit.m* will make a semilog plot of the data contained in the file *countdata.mat*. It will then allow you to choose the desired time interval containing the data to be used in a least-squares fit to a half-life curve (decaying exponential.) The calculated half-life will be displayed and the corresponding curve will be plotted on top of the data. The data points used in the least-squares fit will also be highlighted. Since your data represents a very complex process it is unlikely it will fit a straight line as would a simple exponential decay process. The slope of the curve described by your data at different time intervals will correspond to various decay constants of the decay chain.

The MatLab script *expldata.m* gives an example of how to create the file *countdata.mat* in the correct format for use with *decayfit.m*. The data you will be entering consists of three numbers for each counting interval; the length of the interval, number of counts measured, and the time the interval was started relative to the beginning of the experiment. It should be pretty straightforward to use the *expldata.m* script given to you as a template for entering your own data and creating *countdata.mat*. A more detailed description may be found in *readme6b.html* and in the comments embedded in the scripts.

Simulation of the Radon²²² and Radon²²⁰ (Thoron) Decay Chains

Two other MatLab scripts *radon.m* and *thoron.m* are provided that plot the relative concentrations of Rn²²² and Rn²²⁰ and the nuclei of their respective decay chains as a function of time. The corresponding count rate (re-scaled) is also given as a white curve. These scripts, a readme file and the helper scripts *radonplot.m*, *thoronplot.m* and *colors.m* may be downloaded from the C35 Matlab Scripts World Wide Web

page (<http://www.physics.nwu.edu/C35/PAGES/matscrip.htm>). Some various parameters that you will want to play with are set in the first section of the scripts *radon.m* and *thoron.m* and are described in the script comments. The second section of each script specifies the various decay rates. The final section calculates the initial condition and the resulting decay curves. At the end the corresponding script *thoronplot.m* or *radonplot.m* is called to graph the results. All of this is described in *readme6a.html*.

Use these MatLab scripts to try to answer the following questions.

QUESTIONS

1. Describe the effects of the various daughter nuclei and their half-lives. Compare to the data you took.
2. Which isotope is present in a higher concentration in the air, Rn^{222} or Rn^{220} ?
3. The concentration curves calculated by the MatLab scripts may be selected to start with one of two initial conditions. The first one is simple, at time $t=0$ only Rn^{222} or Rn^{220} exists. None of the daughter nuclei have yet been formed. The second initial condition is the relative equilibrium concentrations of daughter nuclei resulting when the supply of the parent (Rn^{222} or Rn^{220}) is replenished at a constant rate. Which condition do you think more accurately describes the starting condition of your experiment? Discuss. (Hint: Both of these initial condition options may be simulated by toggling the EQUIBCOND parameter between 0 and 1 in *thoron.m/radon.m* MatLab scripts.)

Appendix

RADIOACTIVE DECAY PROCESSES

An atomic nucleus in an excited or unstable state will 'decay' by spitting off some energy in the form of an alpha, beta, or gamma particle. The elemental isotopes with unstable nuclei are called radioactive isotopes. (Each element has a family of isotopes all with the same number of protons but varying numbers of neutrons.) Release of the excess nuclear energy of radioactive isotope proceeds spontaneously as a random event. The exact time that a particular nucleus will shed a particle and decay to a less energetic state cannot be predicted. All that can be foretold is the probability that a certain nucleus will decay during a short interval of time. This probability may be expressed in terms of the mean lifetime τ (the average time that it takes the nucleus to decay) or the decay constant $\lambda=1/\tau$. Over any interval of time Δt that is short compared to the average lifetime ($\Delta t \ll \tau$) the probability that the nucleus will decay is approximately $\Delta t/\tau = \Delta t \lambda$.

As an example suppose a particular radioactive nuclear isotope has a decay constant of $\lambda = 0.02 \text{ sec}^{-1}$ ($\tau = 50 \text{ sec.}$) The probability that an existing nucleus will decay during the next 2 second time period is $(0.02 \text{ sec}^{-1}) \cdot (2.0 \text{ sec}) = 0.04$, i.e. a 4 percent chance. If after 2 seconds it has not yet decayed then it has again another 4 percent chance in the succeeding seconds.

This type of random process is called the exponential process. It is also sometimes called the 'memoryless process' because the length of time a nucleus has existed has no influence on the probability that it will decay in the next Δt seconds. It has no 'memory' of its past history. This means that if a nucleus with an average lifetime of τ_p seconds has not decayed after the first two minutes since it was formed it will at that point still take an average of τ_p more seconds to decay.

For a particular nucleus the decay constant (and therefore the mean lifetime) is a unique value. The value of the decay constant can be calculated based on the availability and kinds of possible states the nucleus may decay into (emission of different types of particle results in various new states.) If there are N possible types of decays for a given nucleus with the probability of each type occurring during a short time interval Δt given by $\Delta t \lambda_1, \Delta t \lambda_2, \dots, \Delta t \lambda_N$ then the overall decay rate is,

$$I_T = I_1 + I_2 + \dots + I_N = \frac{1}{t_1} + \frac{1}{t_1} + \dots + \frac{1}{t_1} = \frac{1}{t_T} .$$

The mean lifetime can be used as an investigative tool to identify a possible source of radiation. Often nuclei may decay into other excited nuclear states in which case subsequent decays will occur through various radioactive daughter products ending only when the nucleus decays to a stable non-radioactive nuclear state.

The number of decaying nuclei per second is proportional to the number that exists in the sample at a given time. The proportionality factor is the decay constant λ_T . This is expressed in equation form by

$$\frac{dN}{dt} = -I_T N .$$

The solution to this equation is the exponential decay equation,

$$N(t) = N_0 e^{-I_T t} ,$$

where N_0 is the number of radioactive nuclei at $t = 0$. An example of this type of decay curve is given in Figure 4.

The number of radioactive nuclei cannot be easily measured, but the rate of decay R may be monitored by counting the number of radioactive particles given off by a sample as a function of time. This decay rate should also show the characteristic exponential decrease as shown in Figure 4.

$$R(t) = -\frac{dN}{dt} = I_T N(t) = I_T N_0 e^{-I_T t} .$$

The rate of nuclear decay of a radioactive isotope is often characterized by its 'half-life' instead of the mean lifetime. Suppose that at time $t = 0$ a sample with a half-life of $T_{1/2}$ contained N_0 nuclei of a radioactive isotope. Then at time $t = T_{1/2}$ half of these nuclei would have decayed, and half the original number of nuclei will be left un-decayed. At time $t = 2T_{1/2}$ half the remaining half of the original number will have decayed away, leaving a quarter of the original sample. At time $t = 3T_{1/2}$, only $1/8$ N_0 nuclei remain un-decayed and so on. A quick calculation relates the decay constant λ_T to the half-life.

Solving

$$\frac{1}{2} N_0 = N_0 e^{-I_T T_{1/2}}$$

for $T_{1/2}$ gives,

$$I_T = \frac{0.693}{T_{1/2}} \quad T_{1/2} = 0.693 t .$$

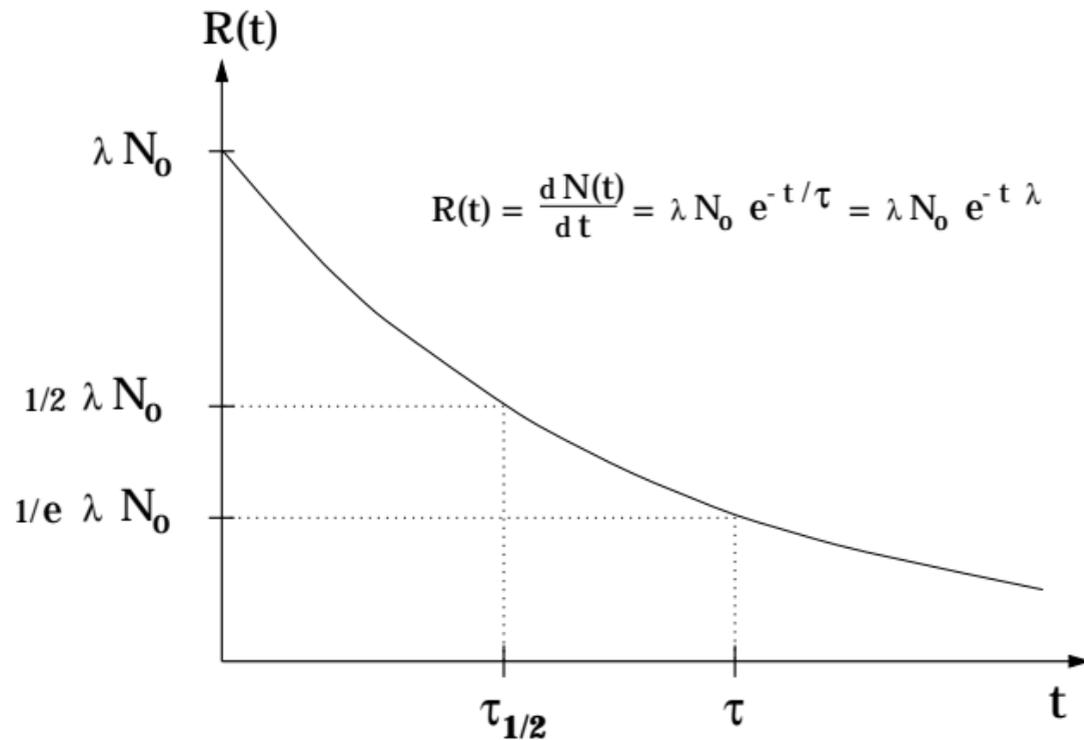
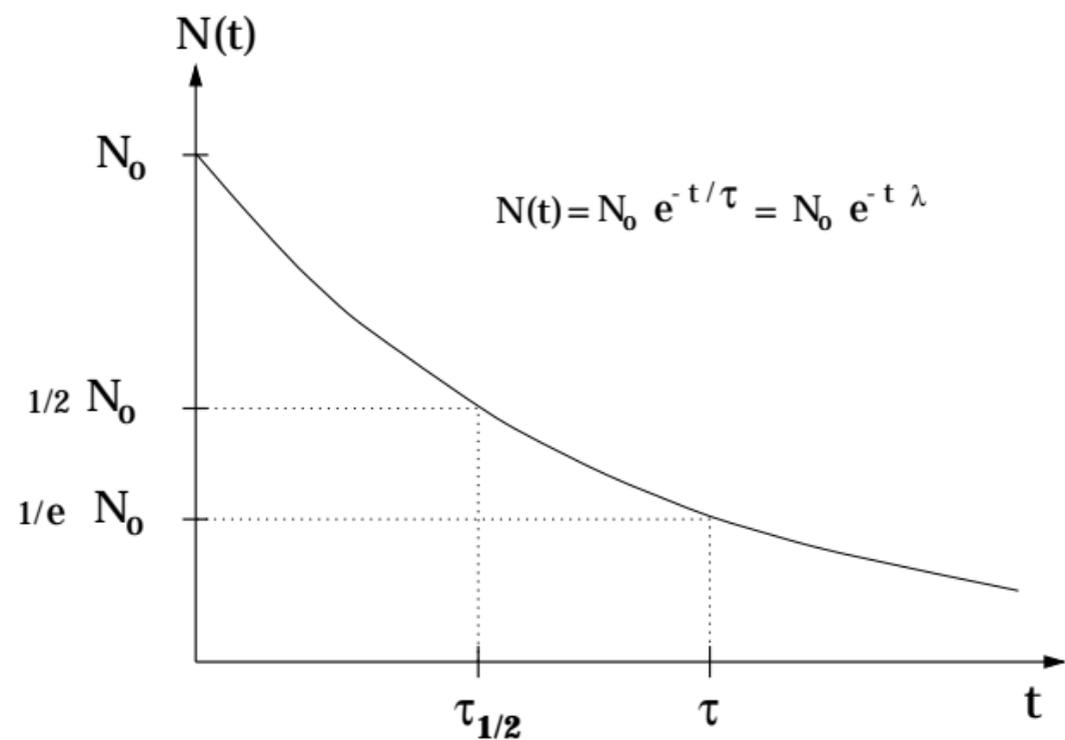


Figure 4 Radioactive Decay and the Corresponding Emission Rate