# PLANCK'S CONSTANT AND
# THE PHOTO-ELECTRIC EFFECT

## INTRODUCTION

In the late 1800's scientists believed that the main principles of physics were well known. There were, however, some phenomena such as *blackbody radiation* and the *photoelectric effect* that had not yet been reconciled in a manner consistent with classical theory. At the turn of the nineteenth century the German physicist Max Planck developed a model explaining blackbody radiation which claimed that the energy of electric oscillations of atoms could only change by discrete prescribed amounts. Einstein later resolved the many puzzling discoveries about the photoelectric effect by making a similar assertion about the energy in a radiation field. The revelation that energy could not be transferred continuously



Figure 1     Blackbody Radiation Spectrum

but must be exchanged in discrete amounts (quanta) led to a radical change in understanding of the physical universe and the development of quantum mechanics.
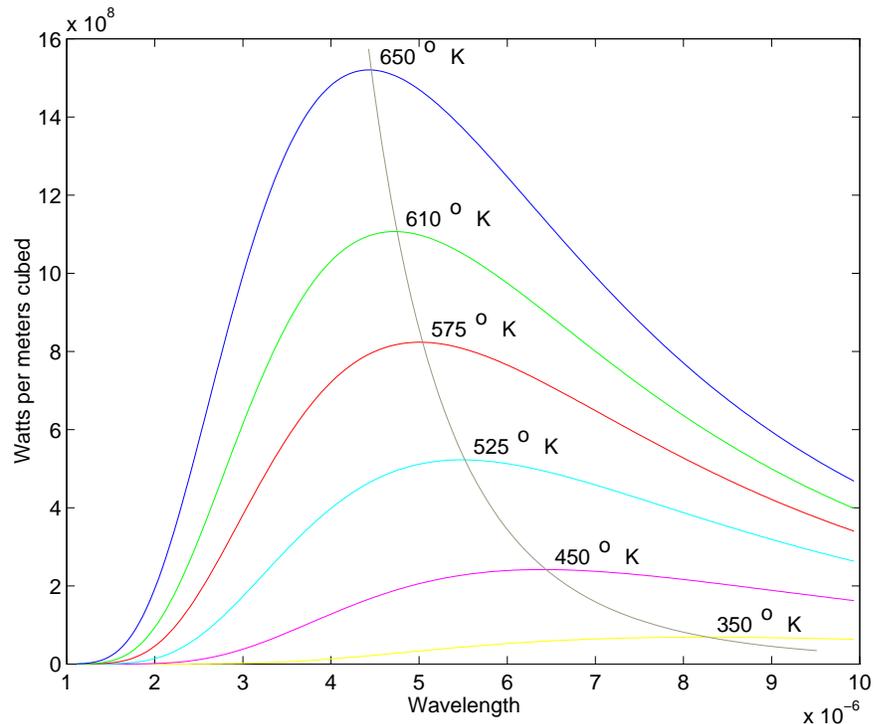
The first sections of this lab will present the modern understanding of blackbody radiation and the photoelectric effect along with the history of how it developed. Then two sets of experiments for investigating the photoelectric effect will be described. The first experiment will make measurements demonstrating Planck's and Einstein's quantum model of radiation. The second experiment will measure Planck's constant, a value which is fundamentally related to the rate and quantum amounts by which energy may be transferred.

## HISTORY AND THEORY

BLACKBODY RADIATION

The term *blackbody radiation* is used to describe the temperature dependent spectrum of radiation emitted by a perfect absorber, that is a body or material that absorbs all radiation incident upon it (see Figure 1.) The radiation spectrum or *spectral distribution* of a source describes the amount of energy given off at each wavelength. A common example closely related to the temperature dependence of the blackbody spectral distribution is the light given off by a piece of heated metal such as the tungsten filament in a light bulb. At ordinary room temperatures most of the radiant energy is emitted at low wavelengths the eye cannot see. As the temperature rises to a couple thousand Kelvin's, light in the infrared and the lower regions of the visible spectrum is given off causing the metal to glow dull red. At higher temperatures the peak spectrum rises and the metal appears yellow then white (at which point all visible wavelengths are strongly present.) The same effect causes variations in flame temperature to appear as changes in color.

Although there is no true "blackbody" it may be closely approximated by a small entrance to an insulated enclosure. Radiation passing through this entrance becomes "lost" in the large space behind with little chance of returning back through the hole. Assuming the inside of the enclosure is in local thermal equilibrium, radiation passing out of the hole will have a spectrum close to that of a theoretical blackbody.

The laws of thermodynamics and Maxwell's equations had been successful in describing several aspects of blackbody radiation. The *Stephen-Boltzmann law* stated that the total radiant flux emitted by a blackbody was proportional to the forth power of the

2

absolute temperature by the *Stephen-Boltzmann* constant $\sigma$ = 5.6686 x $10^{-6}$ (Watts/meters$^2$Kelvins$^4$).

$$I_e = \sigma T^4 \tag{1}$$

A law relating the spectrum peak of blackbody radiation to temperature had also been developed. In 1893 German physicist Wilheim Carl Werner Otto Fritz Franz Wein (known to friends as Willy) presented what is now *Wein's displacement law* .

$$\lambda_{max} T = 2.8978 \text{ x } 10^{-3} \text{ (meters Kelvins)} \tag{2}$$

Attempts on describing the entire form of the blackbody spectrum using classical theory had, however, failed. The *Rayleigh-James law* was based on thermodynamic equilibrium of a blackbody and its environment (energy of radiation absorbed equals energy radiated) and the electromagnetic standing wave modes of a cavity described by Maxwell's equations.

$$\text{spectral distribution = } I(T,\lambda) = \frac{2\pi ckT}{\lambda^4} \quad \text{(Watts/meters}^3) \tag{3}$$

In this equation, developed by Lord Rayleigh and Sir James Jeans, *c* is the speed of light and *k* = 1.380658 x $10^{-23}$ (Joules/Kelvins) is Boltzmann's constant. Their law agrees well with the blackbody spectrum for long wavelengths but fails completely at lower wavelengths. In fact it predicts that the amount of energy radiated increases infinitely as wavelength decreases, a result known as the ultraviolet catastrophe. Similarly, Wilheim Wien's theories correctly described the spectral trends at short wavelengths but failed at longer ones.

In 1900 Max Planck a student of Helmholtz and Kirchoff presented a derivation of a formula that agreed with experimental results at all wavelengths. Starting with an empirical expression that matched the observed data he developed a model that would physically justify it. This model pictured the atoms at the inside surface of the blackbody cavity as electrical oscillators which absorbed and emitted radiation at all frequencies. He then made the important claim that each atomic resonator could only absorb and emit energy in discrete amounts. The amount of energy exchanged in one 'packet' was allowed to be of a value proportional to the oscillator frequency times any integer and the constant *h* (now known as Planck's constant.)

$$\Delta E = m h \nu \quad \text{(Joules)} \tag{4}$$

In equation 4, $m$ is any integer, and $\nu$ is the oscillator frequency. Assuming thermal equilibrium and applying the statistical methods of Boltzmann Planck was able to derive his already empirically determined formula now known as Planck's law.

$$I(T,\lambda) = \frac{2\pi\, h\, c^2}{\lambda^5} \left[ \frac{1}{e^{h\,c\,/\lambda k\,T} - 1} \right] \quad \text{(Watts/meters}^3) \qquad (5)$$

The accepted value of Planck's constant is $(6.6256 \pm 0.0005) \cdot 10^{-34}$ (Joules seconds).

In modern terms we now say that the energy of a given system is quantized meaning that it can take on only a set of discrete values. As a result energy can only be transferred to or from the system in an amount equal to the difference in energy between its current state and one of the possible future energy states. Planck's hypothesis that energy is exchanged only in discrete amounts (quanta) related to the constant $h$, although initially unappreciated is now known to be of greatest importance. The results of his theory are fundamental in describing the interaction between all particles and ultimately the nature of the universe. Planck was awarded a Nobel Prize for his discoveries in 1918.
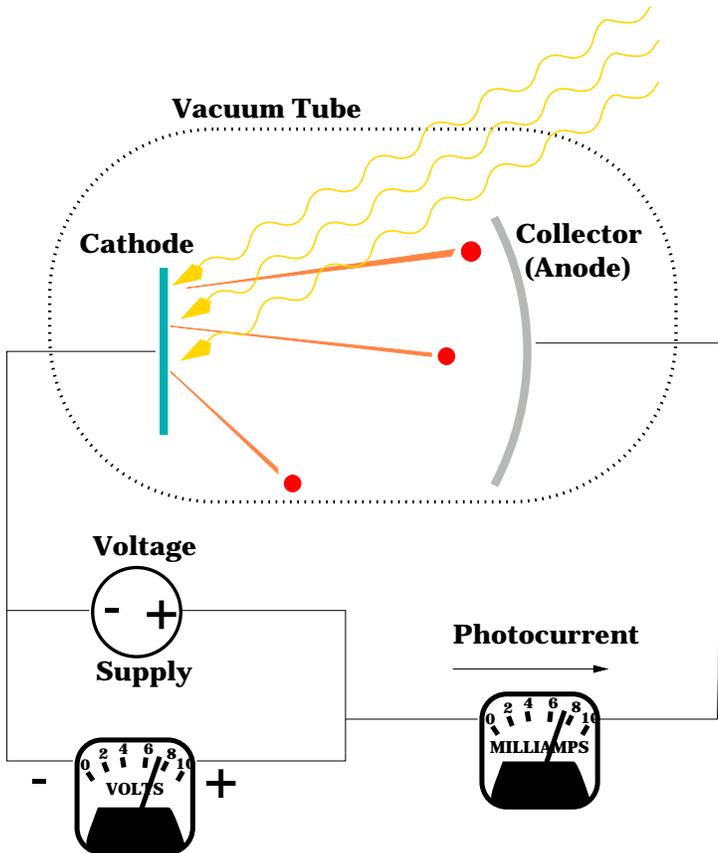


Figure 2   Photoelectric Effect
Light falls on the metal surface and photoelectrons are emitted and collected on the anode, allowing a photocurrent to be measured. As the voltage applied by the voltage source is lowered below the stopping voltage, no photoelectrons will have enough energy to reach the anode and the current will drop to zero.
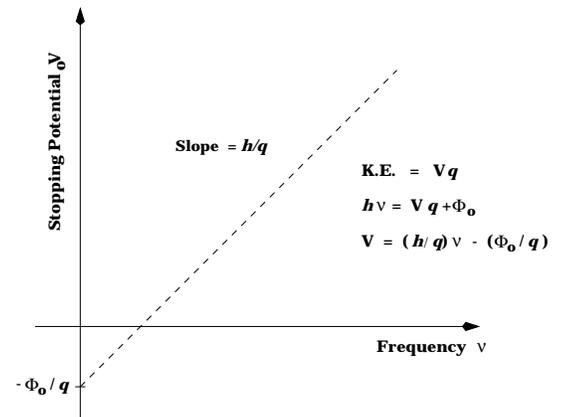


Slope $= h/q$

K.E. $= V q$

$h\nu = V q + \Phi_o$

$V = (h/q)\nu - (\Phi_o/q)$

$-\Phi_o/q$

Figure 3        Photo Current and Stopping Potential

## THE PHOTOELECTRIC EFFECT

During this same time period another unexplained effect was under investigation.  In 1887 during his famous investigations of electromagnetic waves Hertz reported but did not pursue the discovery that the spark induced across a gap between two terminals was stronger when illuminated by ultraviolet light.  Soon after, Wilhelm Hallwachs found in 1889 that negative particles were emitted by illuminated metallic surfaces.  After Anton von Lenard measured the particles to have the same charge to mass ratio as electrons the term *photoelectron* was coined and is still used today for describing electrons emitted by the photoelectric effect.

The emission of electrons by an illuminated material, now termed the *photoelectric effect*, was soon found to have some interesting properties.  By varying the electric potential between a collecting plate and the illuminated metal surfaces  J. Elster and H. Geitel found that electrons were no longer received if the negative retarding voltage was reduced beyond the *stopping potential* $-V_O$ (see Figures 2 and 3.)  By equating the energy needed to reach the collecting plate at this potential, ($qV_O$) with the kinetic energy of the photoelectrons, a maximum energy with which they were emitted was found,

$$K.E. \;\; = \;\; 1/2 \; m_O \; (v_{max})2 \;\; = \; q \; V_O \qquad\qquad (6)$$

where $-q$ and $m_O$ are the charge and mass of an electron. It was also found that the rate of photoelectron emission measured by the *photocurrent* was proportional to the incident irradiance.  Both of these discoveries could be successfully explained by classical theory.  The velocity spectrum could be attributed to the binding energies between the material and the electrons. Photoelectron emission rates proportional to incident energy also follows logically from classical physics.

Further work, however, discovered some properties that were more peculiar.  No delay could be measured between the instant at which the metal surface was illuminated and the start of the emission of photoelectrons.  Surely there must be a finite time needed for the electrons to absorb enough radiant energy to escape the binding energy of the metal surface.  It had also been well documented (although not explained) that the spectrum of the source used had a strong effect on the stopping potential.  Then in 1902 Anton von Lenard made the disturbing discovery that the stopping potential and therefore the maximum velocity of the photoelectrons was independent of the incident radiation flux!  Shouldn't increases in incident radiant

power generate more energetic electrons?  These effects could in no way be reconciled with classical physics.

After several years thought on Planck's hypothesis Albert Einstein solved the photoelectric dilemma in his famous paper of 1905.  He stated that just as the atomic oscillators of a blackbody surface can only change energy in discrete jumps, the energy of an electromagnetic field is also quantized and can only take on a set of discrete values proportional to the field's frequency.  We now call the discrete amounts or quanta of energy which make up an electromagnetic field photons.  Each photon has energy $h\nu$ where $h$ is Planck's constant and $\nu$ is the frequency of the field.  The electrons of a illuminated metal surface absorb energy one photon at a time.  If a photon kicks an electron with enough energy to escape the material's surface it flies away as a photoelectron with kinetic energy equal to the difference between the photon's energy $h\nu$ minus the binding energy $\Phi$.  This result is given by Einstein's photoelectric equation,
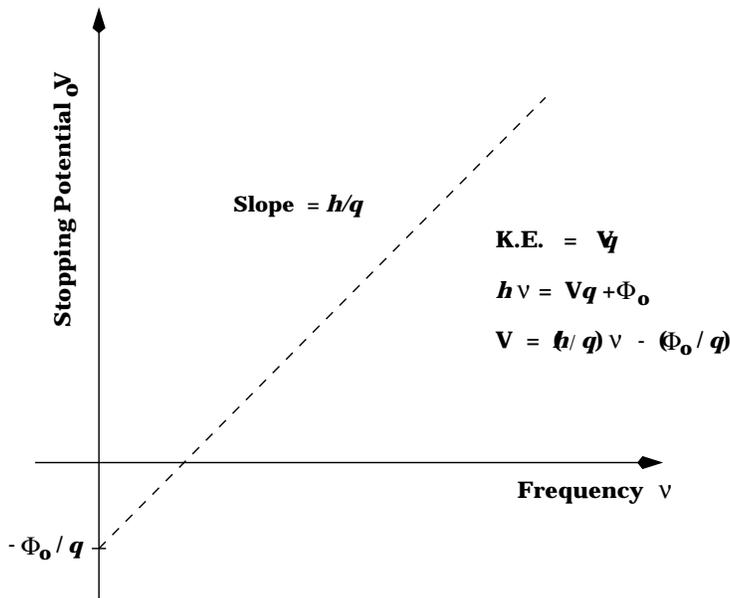
$$\text{K.E.} \ = \ \frac{m\,v^2}{2} \ = \ h\,\nu \ - \ \Phi. \tag{7}$$

When an electron is at the surface and the energy needed to escape takes on the minimum value $\Phi_O$, known as the work function, Einstein's equation becomes,

$$\text{K.E.} \ = \ \frac{m\,v^2}{2} \ = \ h\,\nu \ - \ \Phi_O. \tag{8}$$

This theory neatly accounted for all discrepancies.  Since energy was absorbed in flashes instead of a steady trickle, as soon as the first photons arrived at the surface photoelectrons would begin to appear without delay.  The maximum energy of the photoelectrons (found from the stopping potential) is equal to the energy of the highest frequency photons in the field minus the work function  (see Figure 4).

**Slope** $= h/q$

$\text{K.E.} \ = \ Vq$

$h\nu = Vq + \Phi_o$

$V = (h/q)\nu \ - \ (\Phi_o/q)$

**Frequency** $\nu$

**Stopping Potential** $V$

$-\Phi_o/q$

Figure 4   Stopping  Potential

$$qV_O \ = \ K.E._{max} \ = \ \frac{m \ (v_{max})^2}{2} \ = \ h \ v_{max} \ - \ \Phi_O \qquad (9)$$

Increasing the irradiance increases only the number of photons impinging on the surface and therefore the photocurrent. It is the energy contained by the individual photons that determines the energy of the photoelectrons and therefore the stopping potential.
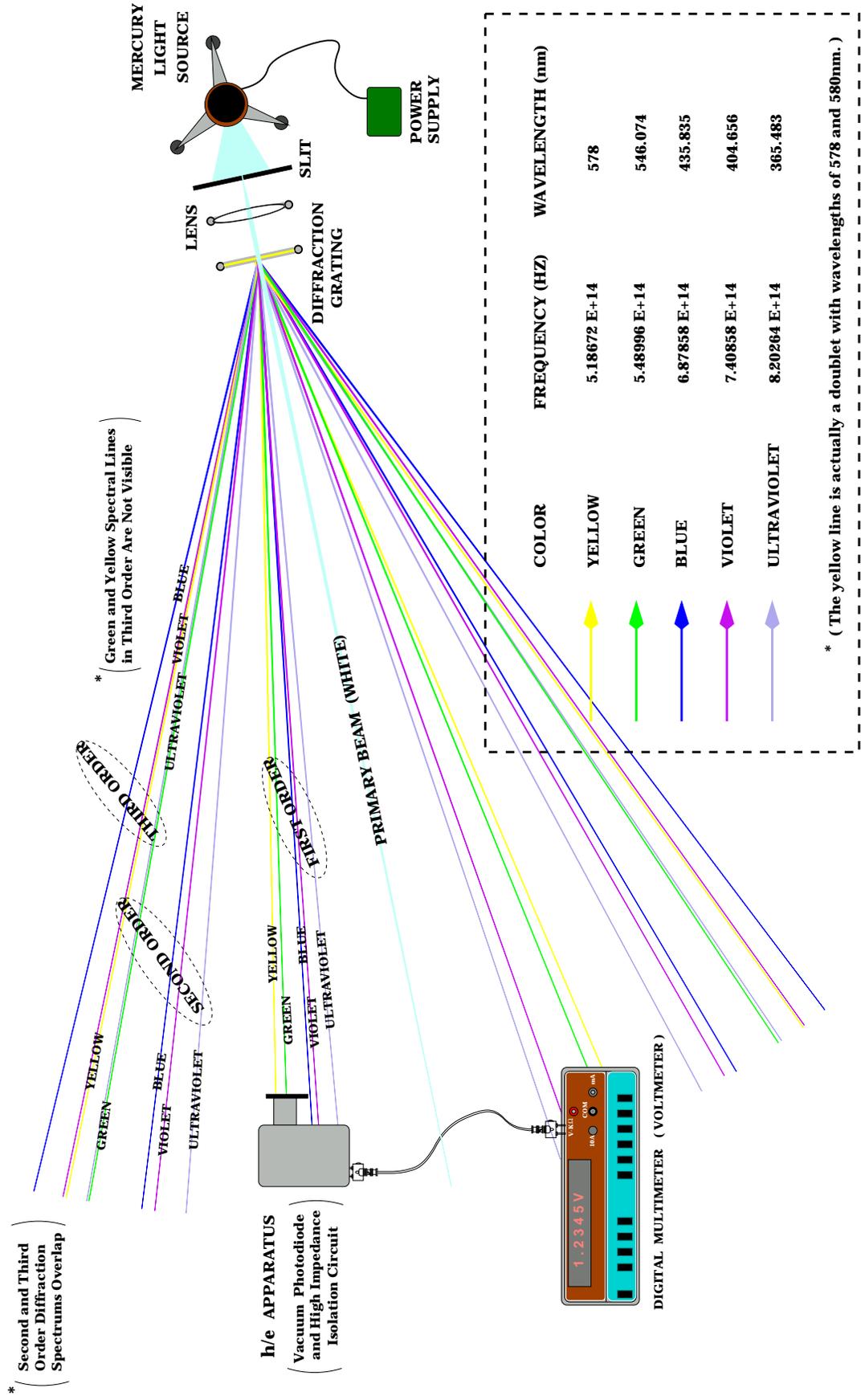
**Figure 5  Photoelectric Experiment  Equipment  Setup**

MERCURY
LIGHT
SOURCE

POWER
SUPPLY

SLIT

LENS

DIFFRACTION
GRATING

* ( Green and Yellow Spectral Lines
    in Third Order Are Not Visible )

BLUE

ULTRAVIOLET  VIOLET

THIRD ORDER

SECOND ORDER

YELLOW

GREEN

BLUE

VIOLET

ULTRAVIOLET

FIRST ORDER

PRIMARY BEAM  (WHITE)

GREEN

YELLOW

BLUE

VIOLET

ULTRAVIOLET

* ( Second and Third
    Order Diffraction
    Spectrums Overlap )

**h/e  APPARATUS**

( Vacuum Photodiode
  and High Impedance
  Isolation Circuit )

DIGITAL  MULTIMETER  ( VOLTMETER )

1. 2 3 4 5 V

V Ω

COM

10 A

mA

| COLOR | | FREQUENCY (HZ) | WAVELENGTH (nm) |
|-------|---|----------------|-----------------|
| YELLOW | | 5.18672 E+14 | 578 |
| GREEN | | 5.48996 E+14 | 546.074 |
| BLUE | | 6.87858 E+14 | 435.835 |
| VIOLET | | 7.40858 E+14 | 404.656 |
| ULTRAVIOLET | | 8.20264 E+14 | 365.483 |

* ( The yellow line is actually a doublet with wavelengths of 578 and 580nm. )

8

>> **DANGER** - THE MERCURY LIGHT SOURCE HAS COMPONENTS IN    <<
>> THE ULTRAVIOLET REGIME WHICH ARE DANGEROUS TO OUR OUR    <<
>> EYES!  DO NOT STARE DIRECTLY INTO IT!  BY PLACING THE    <<
>> SLIT CLOSE IN FRONT OF IT MOST OF THE LIGHT MAY BE    <<
>> BLOCKED.  IT MAY ALSO BE HELPFUL TO PLACE A THIN BOOK    <<
>> OR FOLDER ON TOP OF THE LIGHT SOURCE TO BLOCK THE    <<
>> LIGHT FROM VIEW.

EQUIPMENT LIST

*h/e* Apparatus  (Vacuum Photodiode and High Input Impedance Isolation Circuit)

Mercury Light Source

Diffraction Grating

Lens

Light Filters

Slit

Digital Voltmeter (multimeter)

Timer

Small Lamp

In order to study the photoelectric effect we will use a mercury light source which emits light at several isolated frequencies (see Figure 5.) A thin collimated beam is formed using a slit and a lens (see Figure 5.) This beam is then divided into its separate spectral components using a diffraction grating.  Similar to a prism the grating diffracts the light at different angles depending on wavelength.  Several orders of diffraction may exist for a particular wavelength including the zeroeth order for which the beam passes through
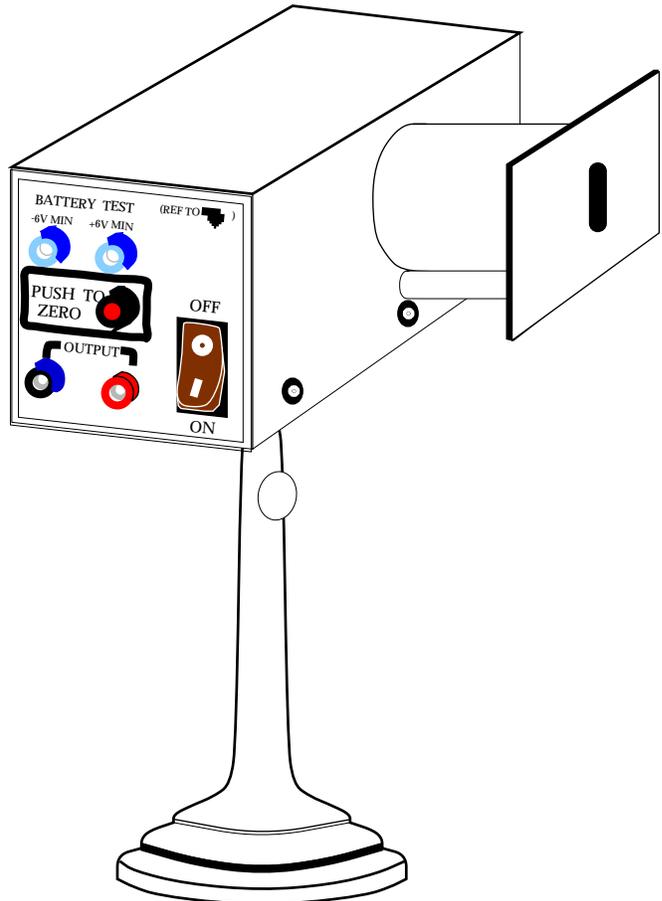


Figure 6    *h/e*  Apparatus

9

undeflected. Finally the beam is aimed onto the vacuum photodiode of the *h/e* apparatus (Figures 5 and 6) producing a voltage difference corresponding to the stopping potential at its output terminals. This may be read by a voltmeter.

Set up the equipment described above as follows:



Figure 7    h/e Apparatus Light Shield

1.  Connect the mercury light source to the circled outlets on its power source, red to red and black Fo black. This source emits at several wavelengths which are given in figure 5.

2.  Place the slit directly in front of the source. Again be careful not to stare into the source directly.

3.  Position the lens so that beam is collimated (neither spreading or narrowing.) You will want to adjust it slightly later to focus the image of the slit as sharply as possible upon the photodiode.

4.  Place the diffraction grating in front of the lens. Placing a hand or a piece of white paper in front of the beam will help to view it more clearly. It will be see that like a prism the grating deflects the different spectral components of the original beam at different angles. It is also seen that the rainbow of lines repeats itself at increasing angles. These are higher orders of the diffraction pattern.

5.  Find the highest frequency (lowest wavelength) component of the mercury light source. It is in the ultraviolet and is not directly visible. However, it may be viewed on most pieces of paper or on the white reflective surface around the entrance to the *h/e* apparatus dues to dyes that fluoresce. It will probably not be seen on your hand.

6.  The *h/e* apparatus has a cylindrical light shield around the entrance on front (see Figure 7.) When making a measurement you will want to close the light shield. This will help to prevent ambient light from corrupting your measurements. You should also turn off and dim as many lights in the room as possible when taking a measurement. Swiveling the light shield of the apparatus out of the way reveals a second aperture with the photo-diode behind it. The actual photodiode entrance may be seen as two white colored windows.

7.  Keeping the light shield open place the *h/e* apparatus a few feet from the mercury source so that one of the spectral lines lands on the aperture on the white reflective surface. Then adjust the position so the beam passes through the first and second apertures and onto the photodiode entrance. Adjust the position of the lens and *h/e* apparatus
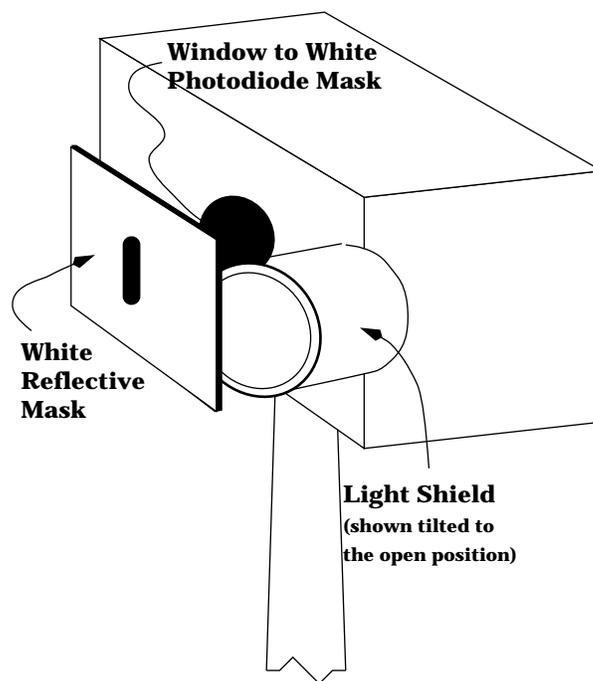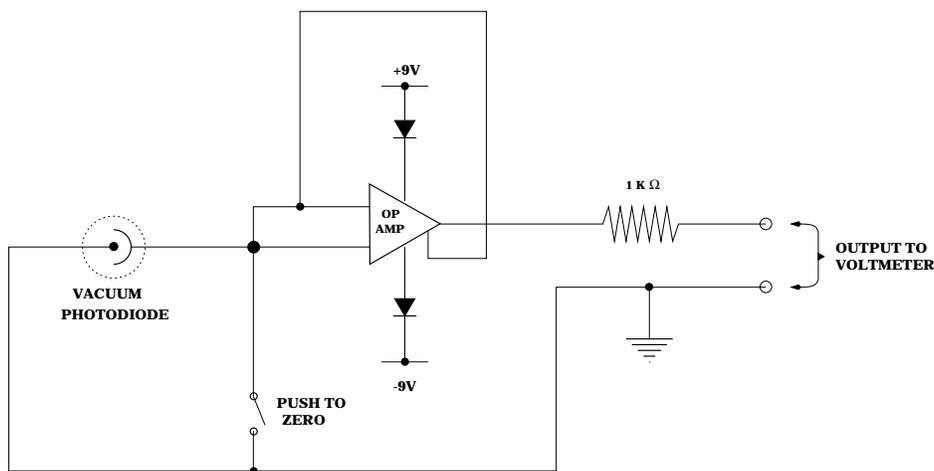
10

Figure 9    *h/e*    Apparatus  Circuit  Schematic



Figure 8   Light  Filters

so that as sharp an image of the slit as
possible is obtained on the photodiode
window.  This will help to make sure that
only one spectral line of the source will be
incident on the photodiode.

7.    Connect the *h/e* apparatus to the voltmeter
(multimeter) using banana connectors.  Ground
should be connected to ground on each device.
The other output terminal of the *h/e* device
should be connected to the voltage measuring
input of the voltmeter.  Make sure the
voltmeter is set to read voltages in the
range of around 1 Volt.

8.    When making measurements with the yellow or
green spectral line the correspondingly colored filter should used (see
Figure 8.)  These should be placed over the hole in the white reflective
mask that forms the entrance to the *h/e*  apparatus. Attach them using
the magnetic strips on the back of the filter.  These filters will block
out frequencies higher than the green or yellow light being measured.
This will help prevent ambient light and ultraviolet light from higher
orders that may overlap the lower diffraction orders from interfering
with the measurements.  The variable transmission filter may be attached
in the same way as the green and yellow filter (see Figure .)

9.    Once all equipment is setup and a spectral line focused onto the
photodiode a measurement of the stopping potential may be read by
pressing the "Press to Zero" button on the *h/e* apparatus.  It will take
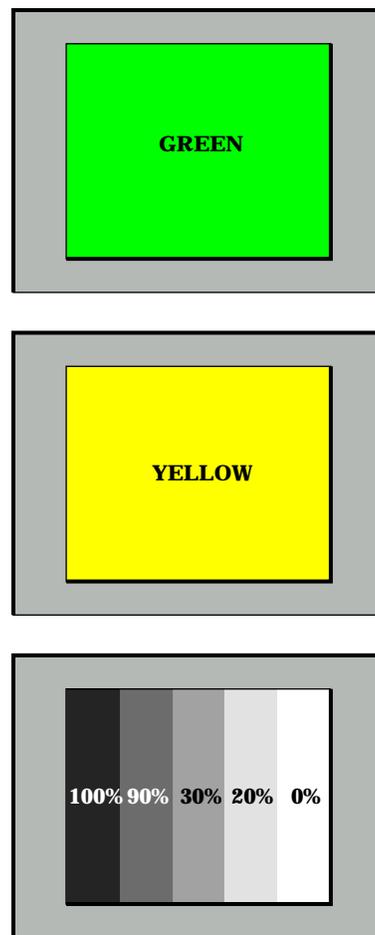around a minute for the voltage to stabilize to the stopping potential.

When monochromatic light (radiation with a extremely narrow frequency spectrum) falls on the cathode plate of the vacuum photodiode, photoelectrons are emitted and collected on the anode. The photodiode has a small capacitance which will be charged by the photocurrent. The growing charge will cause a corresponding retarding potential to build between the anode and cathode. As a result the current will slow as the anode to cathode voltage asymptotically approaches the stopping potential. A material with a very low work function has been chosen for the cathode so that the stopping voltage corresponds very close to the energy of the photons.

$$qV_O \quad = \quad \frac{m \ (v_{max})^2}{2} = h \ v_{max} \quad - \quad \Phi_O \approx \ h \ v_{max} \qquad (10)$$

This voltage cannot be measured directly with a voltmeter because it has a finite impedance and would drain a small current from the capacitance and lower the voltage. Instead a very high impedance amplifier ( $>10^{12} \ \Omega$) with unity amplification ($V_{out} = V_{in}$) is used to isolate the signal from the voltmeter (see Figure 9.)

Note that the time it takes to charge up the capacitance of the photodiode is proportional to the photocurrent and thus the intensity of the light falling on the photodiode.

## EXPERIMENT I - CLASSICAL VS. QUANTUM MODEL

The photon theory of light proposed by Einstein and discussed previously states that the maximum kinetic energy of the photoelectrons is determined solely by the frequency of light and the work function by the equation,

$$qV_O \quad = \quad K.E._{max} \quad = \quad \frac{m \ (v_{max})^2}{2} \quad = \quad h \ v_{max} \quad - \quad \Phi_O. \qquad (11)$$

Thus the higher the frequency of light the more energetic the photoelectrons and the higher the stopping potential. This is in contrast to the classical wave model which predicts that higher intensities with more energetic waves should produce higher energy photoelectrons. The photon or quantum theory predicts that increases in intensity will only increase the number of photoelectrons emitted and thus the

photocurrent.  In part A and B of this experiment we will make measurements that
will help us to see which is the correct model.


EXPERIMENT I  -  PART A

1.  Setup the equipment and align one of the (first order) diffraction lines of
    the frequency of your choice on the photodiode of the $h/e$ apparatus as
    described in the Equipment and Setup Section.  Remember to close the light
    shield, turn off the room lights, and use the correct filter if the green
    or yellow line has been chosen.

2.  Place the Variable Transmission Filter in front of the white reflective
    mask on the $h/e$ apparatus so that the light passes through the clearest
    part of the mask (marked 100%.)  This will allow all of he light to pass
    through.

3.  Press the discharge (Push to Zero) button and use the timer to measure
    approximately how long it takes to charge the capacitance of the photodiode
    and attached circuitry to the maximum (stopping potential).  Due to the
    circuitry you may find that the voltage may lower from a higher voltage to
    the stopping potential instead of rising from zero.  The voltage reaches
    its final value asymptotically (exponential like decay.)  It may take many
    minutes for it to stabilize to a point at which the resolution and noise of
    the voltmeter hides further change.  You will probably need to think a
    little about choosing a criterion for deciding when to stop the timer.

4.  Record the stopping potential.  This should be very close to the energy of
    the photons in the light divided by the charge of an electron.

$$V_O \;=\; \frac{h\ \nu_{max}}{q} \qquad (10)$$

5.  Choose the next section of the variable transmission filter (80%
    transmission) and repeat steps 3 and 4.  Continue making the measurements
    of steps 3 and 4 for the remaining sections of the variable transmission
    filter.

6.  Choose another line/color from the spectrum and repeat the measurements
    above.


EXPERIMENT I  -  PART B

1.  There are five different first order spectral lines formed by the five
    frequencies in the beam of mercury light (Figure 5.)  For each line measure
    the stopping potential.

2.  You may use this data for part of the second experiment also.


EXPERIMENT I  -  ANALYSIS

1.  What effect does varying the intensity of light aimed onto the photodiode
    have on the stopping potential and thus the maximum energy of the
    photoelectrons?

2.  How is the time needed to charge the capacitance up to this potential effected by varying the intensity?  What criterion was used to measure this time?

3.  How is the stopping potential change when the frequency of light is varied?

4.  Describe how these results support the quantum model versus the classical wave model of light.

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│   EXPERIMENT  II  -  MEASUREMENT OF PLANCK'S CONSTANT   │
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The quantum model of light initially developed by Einstein states that the energy of a photon is equal to its frequency times Planck's constant, i.e. $h\nu$.  Examining Einstein's photoelectric equation (equation 9) we see that as a result the stopping potential is proportional to frequency times Planck's constant divided by the charge of an electron.  By measuring the stopping potential at several wavelengths we can use this equation to measure the ratio of Planck's constant to electron charge.

1.  In part Experiment I Part B the stopping potential at is wavelength was measured using the first order spectrum of diffraction lines.  Repeat this same set of measurements for all five spectral components as before but using the second order set of lines.  Note that some of the third order lines overlap the yellow and green lines of the third order set.  When measuring these lines filter out the violet/ultraviolet components using the yellow and green filters.

2.  By plotting the stopping potential for each spectral component as a function of frequency (as in Figure 4) you should get a fairly straight line.  Using the known value of the charge of an electron calculate Planck's constant from the slope of the line.  To be more scientific use a least square error method to calculate the slope/Planck's constant.  This may be easily performed using MatLab or with some calculators.  You can check out Dr Erlenmeyer's Least-Squares Fit web page (http://www.astro.nwu.edu/c35/PAGES/erlenmeyer.html) and the corresponding Theory page (http://www.astro.nwu.edu/c35/PAGES/lsqerr.html) for more details .

3.  Use one of the two methods described in step 2. above to calculate Planck's constant using the data from the first order diffraction lines and the second order lines separately.  The frequencies and wavelengths of each spectral component of the mercury source are given in Figure 5.

3.  How accurate is your result?  What might be some sources of error or bias?

4.  Discuss why the quantum model of light results in a linear relation between stopping potential and the frequency of the light source.

Sections of this writeup were taken from:

<u>Optics</u>, E. Hecht. and A. Zajac; Addison-Wesley Publishing Company

*h/e Apparatus and h/e Apparatus Acessory Kit Manual*, Pasco Scientific Co.

Physics Vol. 2 3rd Edition, Paul Tipler, Worth Publishers