Solutions to Sample Final

1) First we must find the speed of the muons as viewed from Earth. Taking the forward direction of the spaceship to be positive, we have $v_1 = 0.5c$ and $v_2 = -0.9c$. This gives $v = (v_1 + v_2) / (1 + v_1v_2/c^2) = (0.5c - 0.9c) / (1 - 0.5 \times 0.9) = 0.7273c$. The time dilation effect then changes the half-life of the muons as seen from Earth: $\Delta t = \Delta t_0 / (1 - v^2/c^2)^{1/2} = (1.52 \times 10^{-6}\text{ s}) / (1 - 0.7273^2)^{1/2} = 2.21 \times 10^{-6}$ s.

2a) We have $p = m_0v / (1 - v^2/c^2)^{1/2} = (190\text{ kg})(0.28c) / (1 - 0.28^2)^{1/2} = 55.4$ kg-c $= 1.66 \times 10^{10}$ kg m/s.
2b) By conservation of momentum, piece B must have the same momentum as piece A, so:
55.4 kg-c $= m_0(0.6c) / (1 - 0.6^2)^{1/2} = 0.75\, m_0c$, or $m_0 = 55.4\text{ kg} / 0.75 = 73.9$ kg.

3) The ratio of the activities = 0.144 / 0.230 = 0.626 tells us that 0.626 of the original C-14 in the totem is still left. Using the half-life formula yields $0.626 = \exp[(-\ln 2\, (t / 5730\text{ yr})]$. Inverting this, we have $\ln(0.626) = -0.69315\, t / 5730$ yr, or $t = (-0.4684)(5730\text{ yr}) / (-0.69315) = 3872$ yr.

4) The protons and neutrons will fuse to form $^2H_1$, aka heavy hydrogen, aka deuterium. Their masses before the fusion are: proton = 1.007276 u and neutron = 1.008665 u. The mass of the $^2H_1$ is 2.014102 u. The total mass lost in each fusion is $(1.007276 + 1.008665 - 2.014102)\text{u} = (0.001839)(1.66054 \times 10^{-27}\text{ kg}) = 3.054 \times 10^{-30}$ kg. The total energy released by all the protons/neutrons is $E = mc^2 = (6 \times 10^{26})(3.054 \times 10^{-30}\text{ kg})(3 \times 10^8\text{ m/s})^2 = 1.65 \times 10^{14}$ J.

5) We can use $\Delta E\, \Delta t = h / 2\pi$ to estimate the lifetime of the electrons. However, first we must convert the error in wavelength to an error in energy. There are three ways to do this. One way is to compute the energy for the lower side of the line: $E = hc / \lambda = (6.63 \times 10^{-34})(3 \times 10^8)/(519 \times 10^{-9}) = 1.989 \times 10^{-16} / 519 = 3.832 \times 10^{-19}$ J, then do the same for the higher side: $1.989 \times 10^{-16}/ 521 = 3.818 \times 10^{-19}$ J. Subtracting gives $\Delta E = 0.0146 \times 10^{-19}$ J.

Another way is to note that the percentage wavelength error, [1 nm – (–1 nm)] / 520 nm = 2/520 = 0.00385, and the percentage energy error must be equal. So, if $E = hc / \lambda = (6.63 \times 10^{-34})(3 \times 10^8)/(520 \times 10^{-9}) = 3.825 \times 10^{-19}$, then $\Delta E = (0.00385)(3.825 \times 10^{-19}) = 0.0146 \times 10^{-19}$ J, as before.

Last (and probably least), you can compute ΔE directly from $\Delta\lambda$ if you remember your basic calculus:
$\Delta E = \Delta[hc / \lambda] = -hc\, \Delta\lambda / \lambda^2 = (6.63 \times 10^{-34})(3 \times 10^8)(2 \times 10^{-9}) / (520 \times 10^{-9})^2 = 0.0145 \times 10^{-19}$ J, as before.

However you compute ΔE, you have: $\Delta t = (6.63 \times 10^{-34}) / (2\pi)(0.0146 \times 10^{-19}) = 7.5 \times 10^{-14}$ s.

6) An electron accelerated through one volt has 1 eV of energy, so the electron gains 3 keV of energy. The mass of the electron is 511 keV, so the percentage gain is: 3 / 511 = 0.00587 = 0.587%.

7) The distance to the star is contracted in Rachel's frame to: $(14.4\text{ ly})(1 - 0.96^2)^{1/2} = 4.032$ ly. It will therefore take her $t = d / v = 4.032\text{ yr} / 0.96 = 4.2$ years to reach the star.

8) We use the formula for a blackbody, $f_{peak} = (5.88 \times 10^{10}\text{ Hz / K})(6000\text{ K}) = 3.528 \times 10^{14}$ Hz. Then we have $c = f\lambda$ so $\lambda = c / f = (3 \times 10^8) / (3.528 \times 10^{14}) = 850$ nm.

9) $\lambda = h / mv$, so $v = h / m\lambda = (6.63 \times 10^{-34}) / (9.11 \times 10^{-31})(3.8 \times 10^{-7}) = 1.92 \times 10^3$ m/s.

10) $\lambda = h / mv = (6.63 \times 10^{-34}) / (1.008665)(1.66054 \times 10^{-27})(1640) = 2.4 \times 10^{-10}\text{ m} = 0.24$ nm.

11) The number of "m" states for any l is $2l + 1$, so for $l = 8$ we have m = 17.

12) $\lambda = \lambda_{vac} / n$ gives us $n = \lambda_{vac} / \lambda = 469 / 360 = 1.3028$

13) The electron and the positron each have a "mass" of 511 keV, and since they release two identical photons, each photon will also have an energy of 511 keV. $\lambda = hc / E = (6.63 \times 10^{-34})(3 \times 10^{8})/(5.11 \times 10^{5})(1.6 \times 10^{-19})$ $= 2.43 \times 10^{-12}$ m.

14) For hydrogen, we have $E = -13.6 \text{ eV} / n^2$. The energies of the n = 8 and n = 3 levels are therefore $E = -13.6 / 64 = -0.2125$ eV and $E = -13.6 / 9 = -1.5111$ eV, respectively. The electron will give up the energy difference when it changes levels, or $-0.2125 - (-1.5111) = 1.3$ eV. Converting this into a wavelength: $E = hc / \lambda \rightarrow \lambda = hc / E = (6.63 \times 10^{-34})(3 \times 10^{8}) / (1.3)(1.6 \times 10^{-19}) = 9.56 \times 10^{-7}$ m.

15) From the data sheet, we have masses of $^{4}He = 4.002603$ u, $^{226}Ra = 226.025406$ u, $^{222}Rn = 222.017574$ u. So, in the decay there is a mass loss of: $(226.025406 - 222.017574 - 4.002603)\text{u} = (0.005229)(931.58 \text{ MeV})$ $= 4.87$ MeV. This energy will show up as kinetic energy for the α-particle.

16) The tuning fork is vibrating at half the frequency it should, so it is experiencing a time dilation of two. Thus the mass must also be dilated by two. Answer: B

17) The monitor gains energy (from the electricity) and thus gains mass. Answer: E

18) $E = hf = hc / \lambda$, so I need to decrease the wavelength to increase the energy. Answer: B

19) Answer: E

20) Answer: C

21) The mass of the uranium decreases by 235 – 227 = 8 in the decay to thorium, so we must emit two α-particles to shed this mass. Since only answer E has two α-particles, we could stop now. However, just to be sure, we note that the two α-decays will reduce the number of protons in the uranium to 92 – 2 – 2 = 88, which is two shy of the 90 needed to make thorium. Since β-decays convert neutrons to protons, we also need two β-decays to complete the decay chain. Answer: E

22) From $d \sin\theta = m\lambda$, the largest θ corresponds to the largest λ, which is red light. Answer: A

23) β-decay causes the atomic number to rise, not fall, so Answer C is impossible.