Solutions to Physics 130-3 Final, Summer 2009

1) (10 points) Scientists at Physicsland Labs are studying unstable particles called zenons. They see that just-created zenons emerge from the accelerator at 0.99 c and travel an average of 28 m across the lab before disintegrating. What would be the half-life of a zenon if it were at rest in the lab?

Solution: The average time the zenons "live" in the lab is $t = d/v = 28/(0.99)(3 \times 10^8) = 9.428 \times 10^{-8}$ s. This time is dilated relative to the moving zenons. We have $\gamma = (1 - 0.99^2)^{1/2} = 0.141$, so the half-life in the rest frame must be <u>shorter</u> by this amount, or $t = (0.141)(9.428 \times 10^{-8}) = 1.33 \times 10^{-8}$ sec.

2) The Bohm-Aharonov Effect is an exotic quantum phenomena which results when an electron is diffracted around a magnetic field. When I was in graduate school, another student asked my advice about measuring this effect using a clunky old magnet that he'd found in a storage closet.

a) (5 points) His magnet was probably 5 cm in diameter. Estimate the maximum kinetic energy (in joules) that an electron can have if it is to experience any significant degree of diffraction around an object of this size. State *very briefly* how you are making your estimate.

Solution: To show significant diffraction, the electron's wavelength should be roughly as large as the cylinder. We estimate λ to be 5 cm. At this point a number of students used $E = hc/\lambda$ to relate λ to E, but this formula is only good for photons (i.e., zero rest mass). For electrons we have $\lambda = h/p = h/mv$, so $v = h/m\lambda$ and thus $E = ½ mv^2 = h^2 / 2\lambda^2 m = (6.63 \times 10^{-34})^2 / (2)(0.05)^2(9.11 \times 10^{-31}) = 9.65 \times 10^{-35}$ J

2b) (3 points) Assume that the inside of your vacuum chamber for measuring the Bohm-Aharonov Effect is sealed off from all light, radio waves, or other external radiation. However, the chamber is at room temperature, or 290 K. What would be the most predominant energy of the remaining photons?

Solution: The peak in the blackbody radiation of an object at 290 K occurs at $\lambda = 2.90 \times 10^{-3} / T = 2.90 \times 10^{-3}/290 = 10^{-5}$ m. The photon energy is $E = hc/\lambda = (6.63 \times 10^{-34})(3 \times 10^8)/(10^{-5}) = 1.99 \times 10^{-20}$ J.

2c) (2 points) Using your answers to Parts 2a and 2b, give one specific reason why I fell on the floor laughing after the student left my lab.

Solution: The ratio between the electron energy and the energy of the infrared photons is an incredible $(1.99 \times 10^{-20})/(9.65 \times 10^{-35}) = 2.06 \times 10^{14}$! Interaction with even one photon would blast the electron to an energy 100 billion times too hot to diffract. Also, if you calculate the velocity of the electron using $v = h/ml = (6.63 \times 10^{-34})/ (9.11 \times 10^{-31})(0.05) = 0.0146$ m/s, you can see that it would take the electron $0.0146^{-1} = 68.5$ seconds to travel one meter. But in that time, gravity would cause the electron to fall $d = ½ gt^2 = 0.5(9.8)(68.5)^2 = 22{,}987$ meters, or about 14 miles! Good luck hitting the target.

3) You are standing on the ground as two very fast rockets speed towards each other. Both rockets are moving at 0.6c relative to you. They are initially 3000 km apart, relative to you.

a) (3 points) How long will it take them to collide from your point of view?

Solution: They will collide when either rocket reaches the center (3000 / 2 = 1500 km). We have $t = d/v = (1500 \times 10^3) / (0.6)(3 \times 10^8) = 0.0083$ sec.

3b) (3 points) From your point of view, how much time will run off the watch of the pilot of Rocket 1 as he covers the distance to the collision point?

Solution: We have $\gamma = [1 - 0.6^2]^{1/2} = 0.8$, and the 0.0083 sec in Part (a) is from *your* point of view. Thus the pilot's watch is the slow one (<u>less</u> time must pass), so $t = (0.8)(0.0083) = 0.0067$ sec.

3c) (3 points) Initially, how far is Rocket 2 from Rocket 1, according to the pilot of Rocket 1?
Solution: The distance will be length-contracted by $\gamma = [1 - 0.6^2]^{1/2} = 0.8$ according to pilot 1. So, $d = (3000)(0.8) = 2400$ km.

3d) (3 points) How long will it take the rockets to collide, according to the pilot of Rocket 1?
Solution: A number of students were confused about the distance for this part. 2400 km is distance separating the two spacecraft, from the viewpoint of the pilot. Since the other spacecraft is at the other end of the full 2400 km, you need to use the full 2400 km, not half of it, to calculate the speed. Relative to himself, pilot 1 is standing still. Relative to pilot 1, Rocket 2 is approaching at a speed of $v = (0.6c + 0.6c)/(1 + 0.6^2) = 0.882\ c = 2.65 \times 10^8$ m/s. Rocket 2 will cover the distance between them in $t = d/v = (2400 \times 10^3)/(2.65 \times 10^8) = 0.00907$ sec.

4) (8 points) A stationary atom of $_{239}Pu^{94}$ is struck by a neutron carrying 8 MeV of kinetic energy. The plutonium absorbs the neutron, goes into convulsions, then disintegrates into an atom of $_{140}Ba^{56}$, an atom of $_{96}Sr^{38}$, and four neutrons. What is the kinetic energy of the reaction products after the disintegration?
Solution: The mass of the plutonium is 239.0521565 u, and the total mass that it disintegrates into is 139.910599 + 95.921680 + 3 x 1.00866 = 238.858259 u. The total energy released is therefore $E = mc^2 = (239.0521565 - 238.858259)(1.660538782 \times 10^{-27})(3 \times 10^8)^2 = 2.89777 \times 10^{-11}$ J $= 1.81 \times 10^8$ eV = 181 MeV. The total kinetic energy is 181 + 8 = 189 MeV.

5) (10 points) Natural uranium consists of 99.7% $_{238}U^{92}$ and 0.3% $_{235}U^{92}$. Given a half-life of 4.468×10^9 years for U-238 and 7.038×10^8 years for U-235, what would have been the relative percentages of U-238 and U-235 in natural uranium 2.5 billion years ago?
Solution: $N(t) = N_o \exp(-t \ln 2/\tau)$ means $N_o = N_{now} \exp(t \ln 2/\tau)$. Using billions of years as our time unit, we see that every 99.7 atoms of U-238 existing now corresponds to $N_o = 99.7 \exp(2.5 \ln 2 / 4.468) =$ 146.9 atoms in 2.5 billion BC. Likewise for U-235, $N_o = 0.3 \exp(2.5 \ln 2 / 0.7038) = 3.5$ atoms then. The abundance of U-235 was 3.5/(146.9 + 3.5) = 2.3%. The U-238 made up 100 – 2.3 = 97.7%

6) (5 points) If an 80-kg person accidentally ingests 10 μCi of an isotope that radiates 5 MeV β-particles, how many rems will they have absorbed by the end of one week? You may assume that the half-life of the isotope is very, very long compared to one week.
Solution: $10\ \mu Ci = (10)(10^{-6})(3.7 \times 10^{10}$ decay/s$) = 3.7 \times 10^5$ decay/s. The β-particle energy is 5×10^6 eV, so the energy absorbed/week is $E = (60 \times 60 \times 24 \times 7)(3.7 \times 10^5)(5 \times 10^6) = 1.12 \times 10^{18}$ eV = 0.179 J. Using a quality factor of 2 for the β's, we have: rem = (100)(2)(0.179)/80 = 0.448

7) (5 points) The planet Jupiter is the most massive in the solar system, but it is cold. It has a radius of 69,911 km and a mean surface temperature of 110 K. The Earth has a radius of 6371 km and a mean surface temperature of 254 K. If we make the approximation that both planets have uniform surface temperatures, which planet radiates the most heat into space, and by what ratio over the other?
Solution: The power radiated by a blackbody is $P = \sigma AT^4$, so the ratio of the power radiated by Jupiter as compared to the Earth is $P_J / P_E = (69{,}911 / 6371)^2 (110 / 254)^4 = 4.24$

Multiple Choice (3 points each)

8) Which of the following items had little or nothing to do with experimentally verifying Einstein's theory of relativity?

- A) An experiment that flew atomic clocks around the world
- B) The lifetime of muons created in the Earth's atmosphere
- C) The anomalous precession of Mercury's perihelion
- D) The search for an ether wind passing Earth
- E) The bending of light as it passes the Sun
- F) The discovery of the anti-electron

Answer: F

9) Suppose I have a tube of hydrogen gas, and I run a weak current through it which is only capable of exciting electrons into the n = 2 energy level.

9a) How many spectral lines would this tube give off, according to the simple Bohr model?

A) 1 B) 2 C) 3 D) 4 E) 5 F) 6 G) 7

Answer: A The only possible transition is 2 to 1.

9b) Let us assume that the "*l* " and "s" quantum numbers have NO effect on the energy of a quantum state. According to modern quantum mechanics, how many spectral lines would the tube give off if I placed it in a strong magnetic field?

A) 1 B) 2 C) 3 D) 4 E) 5 F) 6 G) 7

Answer: F The magnetic field will cause the n=2, *l*=1, m= –1, 0, 1 states to split into three different energies. (The *l*=0 levels for both n=1, n=2 are unaffected, because m=0 is the only value for *l*=0.) This means that the spectral line in Part 9a will split into three lines. There can also be transitions *between* the m-levels (i.e., n=2, *l*=1, m=0 could transition to n=2, *l*=1, m=1), which adds three more lines (m=1 to m=0, m=1 to m= –1, m=0 to m= –1). I gave two points if you answered **C**.

10) Which of the following statements about proton-proton fusion is *not* correct?

A) it is the source of the Sun's energy
B) it has never been achieved on Earth
C) it is mediated by the weak nuclear force
D) it produces the energy in a hydrogen bomb
E) the fusion product is deuterium (^2H)
G) it emits neutrinos

Answer: D The fusion process in a hydrogen bomb is caused by tritium-deuterium ($H^3 – H^2$) fusion, not proton-proton.

11) You are testing a material for possible use in solar cells. You find that it will *just barely* make a voltmeter twitch if you shine yellow light (λ = 500 nm) on it. What would be the maximum voltage of the current produced if you were to shine UV light (λ = 300 nm) on it?

A) 2.30 volts B) 1.66 volts C) 1.38 volts D) 4.13 volts
E) 6.43 volts F) 0.85 volts G) 5.17 volts H) 13.61 volts

Answer: B The work function of the material is E = 1243 / 500 = 2.486 eV. The energy of the UV light is E = 1243 / 300 = 4.143 eV. The maximum energy photo-electron that the UV can create from this material is thus 4.143 – 2.486 = 1.66 eV.