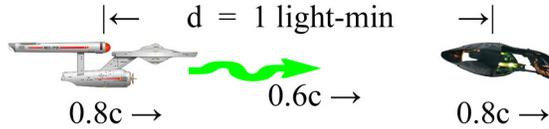


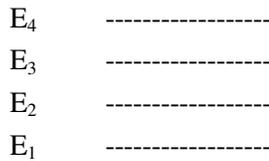
Old Homework Problems from the Vault

1) The Starship Enterprise is following a Klingon Warbird. Both spaceships are moving at exactly $0.8c$ with respect to Starbase 3108, where you are monitoring the situation. The two ships are one light-minute apart, also as seen from Starbase 3108. The Enterprise fires a phaser blast directly forward which leaves the Enterprise at $0.6c$ as seen from the Enterprise.



- 1a) How fast is the phaser blast moving with respect to you?
- 1b) How long will it take the phaser blast to reach the Klingons, from your point of view?
- 1c) How long will it take the phaser blast to reach the Klingons, from their point of view?

2) Consider the energy-level diagram below for a hypothetical atom:



where $E_1 = 0$ eV (ground state), $E_2 = 4$ eV, $E_3 = 7$ eV, and $E_4 = 9$ eV.

- 2a) How many spectral lines will this atom have? Show how you counted them.
- 2b) What is the shortest wavelength of light (in meters) that this atom can emit? The longest?

3) Radon gas is created by the α -decay of radium. The atomic masses of the relevant nuclei are:

$$\begin{aligned} \text{radium} &= 226.0254 \text{ u} \\ \text{helium} &= 4.0026 \text{ u} \\ \text{radon} &= 222.0176 \text{ u} \end{aligned} \quad \text{where } u = 1.66054 \times 10^{-27} \text{ kg.}$$

What is the velocity of the α -particle (in meters per second) as it moves away from the radon? It is OK to assume that the radon atom remains almost stationary, i.e., that it effectively has no kinetic energy.

4) Medieval alchemists spent a great deal of time searching for the Philosopher's Stone, which was a magical substance that would change a base metal such as lead into gold. Construct a possible chain of α -decays and β -decays that could turn lead (${}^{208}_{82}\text{Pb}$) into gold (${}^{196}_{79}\text{Au}$). Never mind that your hypothetical α - and β -decays are probably not found in nature.

5) Suppose I accelerate a positron in a particle accelerator until it has a kinetic energy of exactly 511 keV.

- 5a) What is its velocity according to Newton? (give % of lightspeed)
- 5b) What is its velocity according to Einstein? (give % of lightspeed)
- 5c) Suppose I let the positron collide with a stationary electron. What is the wavelength of the most powerful photon that this collision could possibly produce?

Solutions

1a) Since the Enterprise is moving at $0.8c$ relative to you, but the phaser blast is moving at $0.6c$ relative to the Enterprise, we have to use the relativistic velocity addition formula: $v_{\text{net}} = (v_1 + v_2)/(1 + v_1 v_2/c^2)$. Setting $v_1 = 0.8c$ and $v_2 = 0.6c$ gives $v_{\text{net}} = (0.8c + 0.6c)/(1 + 0.48) = 0.946c$

1b) From your point of view, the two spacecraft are one light-minute apart, and the phaser blast is moving at $0.946c$. The time will be: $t = d/v = 60 \text{ light-seconds} / 0.946c = 63.4 \text{ seconds}$.

1c) The distance between the ships is one light-minute in your frame. This is a *contracted* distance, because the ships are moving relative to you. We need the *uncontracted* distance in the Klingons' rest frame. We can use $L = L_0(1 - v^2/c^2)^{0.5}$, where $L = 60 \text{ light-sec}$ and L_0 is what we want. We have: $L_0 = (60 \text{ light-sec})/(1 - 0.64)^{0.5} = 100 \text{ light-sec}$. Since the Klingon ship and the Enterprise are not moving relative to each other, the Klingons see the phaser blast moving at $0.6c$, so we have: $t = d/v = 100 \text{ light-sec} / 0.6c = 167 \text{ sec}$.

2a) There is one spectral line for each possible transition. We have: $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$, and $2 \rightarrow 1$. There are six lines.

2b) The shortest wavelength will correspond to the highest-energy jump, because $E = hc/\lambda$. The highest-energy jump is the one going all the way from the top to the bottom, E_4 to E_1 , which is 9 eV . We have: $\lambda = hc/E = (6.63 \times 10^{-34})(3 \times 10^8) / 9(1.6 \times 10^{-19}) = 1.38 \times 10^{-7} \text{ m}$. Similarly, the longest wavelength will correspond to the lowest-energy jump. Looking at the numbers, you can see that the E_4 to E_3 jump ($= 2 \text{ eV}$) is the smallest one. We have: $(6.63 \times 10^{-34})(3 \times 10^8) / 2(1.6 \times 10^{-19}) = 6.21 \times 10^{-7} \text{ m}$.

3) The radium mass = 226.0254 u . After the decay, the radon mass plus the alpha = $222.0176 \text{ u} + 4.0026 \text{ u} = 226.0202 \text{ u}$. This means that $226.0254 \text{ u} - 226.0202 \text{ u} = 0.0052 \text{ u}$ of mass has "disappeared". Of course it has been turned into energy, so we have: $E = mc^2 = (0.0052)(1.66054 \times 10^{-27} \text{ kg})(3 \times 10^8 \text{ m/s})^2 = 7.77 \times 10^{-13} \text{ J}$. Since we have assumed that the radon atom is not moving, all of this released energy must show up as kinetic energy in the α -particle. To find the velocity we can use the classical formula $E = \frac{1}{2}mv^2$, or $v^2 = (2E/m)$. $v = [2(7.77 \times 10^{-13} \text{ J})/(4.0026)(1.66054 \times 10^{-27} \text{ kg})]^{0.5} = 1.53 \times 10^7 \text{ m/s}$.

4) To get to the correct atomic weight, we can use a series of α -decays. You lose four units for each decay, so we need: $(208 - 196) / 4 = \text{three } \alpha\text{-decays}$. You also lose two in the atomic number for each α -decay, so after three of them our atomic number is: $82 - 3 \times 2 = 76$. To get to an atomic number of 79 we need to convert three neutrons into protons, which means we will need three β -decays. So, any combination of three α -decays and three β -decays will turn lead into gold.

5a) Newton would use the formula $E = \frac{1}{2}mv^2$, or $v = (2E/m)^{0.5}$. Putting in all the numbers gives us: $v = [2(5.11 \times 10^5)(1.6 \times 10^{-19})/(9.11 \times 10^{-31})]^{0.5} = 4.24 \times 10^8$, or $1.41c$ (Oops. Looks like the classical formula is not very accurate at these speeds.)

5b) The kinetic energy of the positron is 511 keV , and since $E = mc^2$ this means that the total mass of the positron from your viewpoint is $511 \text{ keV (kinetic)} + 511 \text{ keV (rest mass)} = 2(511 \text{ keV})$. The formula for relativistic total mass, $m = m_0/(1 - v^2/c^2)^{0.5}$, then allows us to say: $2(511 \text{ keV}) = 511 \text{ keV}/(1 - v^2/c^2)^{0.5}$ or $2 = 1/(1 - v^2/c^2)^{0.5}$. Solving for v/c gives us: $v/c = (1 - \frac{1}{4})^{0.5} = 0.866$

One could also use the formula for relativistic kinetic energy and grind through the algebra, but that's the hard way. Note that keeping the energies and masses in units of keV means you can avoid tedious units conversions.

5c) The positron and the electron will annihilate upon contact, converting their rest masses into energy. Also, the positron is carrying 511 keV of kinetic energy and that has to go somewhere. So the maximum energy available for a photon is: $3(511 \text{ keV}) = (1533 \times 10^3)(1.6 \times 10^{-19}) = 2.45 \times 10^{-13} \text{ J}$. From $E = hc/\lambda$ we have:
 $\lambda = hc/E = (6.63 \times 10^{-34})(3 \times 10^8) / (2.45 \times 10^{-13}) = 8.12 \times 10^{-13} \text{ m}$.