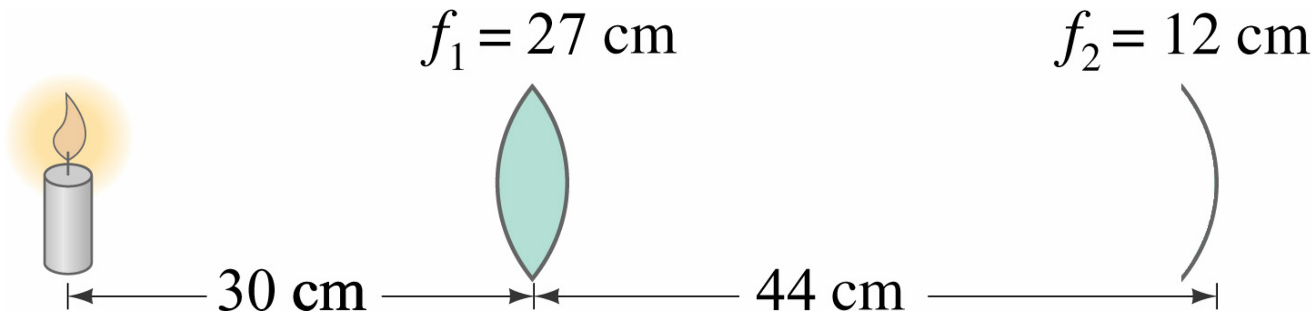
Lens Problems with Detailed Explanation

A candle is setting in front of a convex lens and a concave mirror as shown below.
a) Where will the image of the candle be (relative to the real candle)?
b) What is the magnification of the image?
c) Is the image upright or inverted? Real or virtual?



Solution:

a) First we need to calculate where the image from the lens is. Since the object in this case is actually an object, its distance from the convex lens must be positive. In the diagram above the candle is to the left of the lens, so the left side is positive here – but if I flipped the diagram around and put the candle on the right-hand side, then *that* would be the positive side. The positive side for the object of a lens is whatever side the light is coming from.

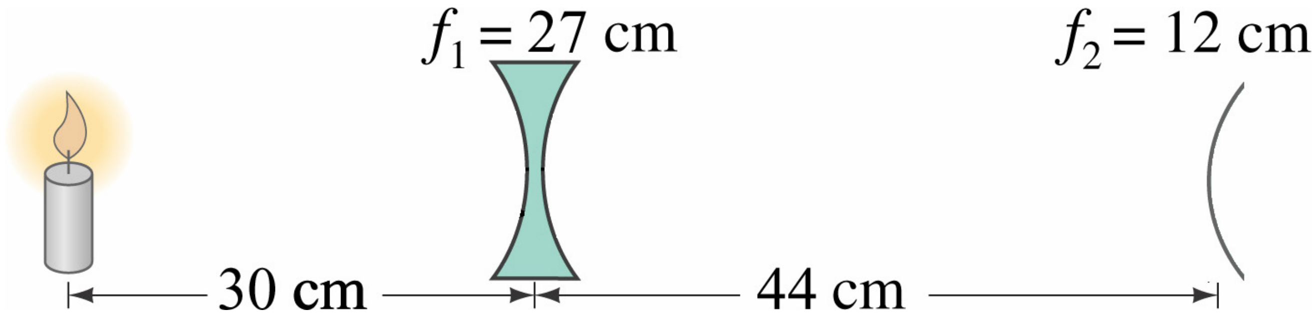
By definition, the focal length of a convex lens is always positive, so we have: $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$
or $1/d_I = (1/27 - 1/30) = 0.0037$, or $d_I = 270$ cm. This image distance is positive, so the image is to the *right* of the convex lens. (Remember, the sign convention for convex lenses says that the positive side for the image is *opposite* the side where the light is coming from.)

Next, we use this image as the object for the mirror. The image is 270 cm to the right of the lens, so that means it is 270 cm – 44 cm = 226 cm to the right of the mirror. An object *behind* the reflecting surface of a mirror has a negative distance (this can only happen if the "object" is actually an image from another optical component), so $d_O = -226$ cm for the mirror. By definition, the focal length for a concave mirror is always positive, so this gives us: $1/d_I = 1/f - 1/d_O = [1/12 - 1/(-226)]$
$= (0.0833 + 0.0044) = 0.0877$, or $d_I = 11.4$ cm. The image distance is positive, which means it is in front of the reflecting surface, that is, to the *left* of the mirror. (Both objects and images are positive if they are in front of the mirror.) So, the final image is (30 cm + 44 cm – 11.4 cm) = 62.6 cm to the right of the candle.

b) The overall magnification is the product of the magnifications of each optical component. Using the formula: $m = -d_I / d_O$, we have for the convex lens: $m = -(270\text{ cm})/(30\text{ cm}) = -9$.
Likewise for the mirror we have: $m = -(11.4\text{ cm})/(-226\text{ cm}) = 0.05$, so the overall magnification is: $(-9)(0.05) = -0.45$

c) The overall magnification is negative, so the image is inverted. The image is in front of the silvered surface of the mirror, so a screen could be placed there onto which to "project" the image, thus the image is real.

Now let's do the problem again, except this time with a *concave* lens and a *convex* mirror, exactly the reverse of above.



a) As before, the object distance for the lens is 30 cm, but now we have a negative focal length because the lens is concave. This gives us: $1/d_I = 1/f - 1/d_O = (-1/27 - 1/30) = -0.0704$, or $d_I = -14.2$ cm. The negative sign means that the image is to the *left* of the lens.

Thus the image is 44 cm + 14.2 cm = 58.2 cm to the left of the mirror. The object distance for the mirror is therefore positive, since the object is on the reflecting side of the mirror. However, the focal length for a convex mirror is alway negative. This gives us:
$1/d_I = 1/f - 1/d_O = (-1/12 - 1/58.2) = -0.0833 - 0.0172 = -0.1005$, or $d_I = -10$ cm. The negative sign means it is behind the reflecting surface, that is, to the right of the mirror.

The final image is: 30 cm + 44 cm + 10 cm = 84 cm to the right of the candle.

b) The magnification for the lens is: $-(-14.2) / 30 = 0.473$
The magnification for the mirror is: $-(-10) / 58.2 = 0.172$
The overall magnification is: $(0.473)(0.172) = 0.0813$ .. .. .. or perhaps I should say de-magnification, since this system reduces the image size by about a factor of 12.

c) The magnification is positive, so the image is upright. The image is behind the mirror, so it cannot be projected, thus it is virtual.