46. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know \( C_1 \) and \( C_2 \). From Eq. 25-9,

\[
C_2 = \frac{\varepsilon_0 A}{d} = 2.21 \times 10^{-11} \text{ F},
\]
and from Eq. 25-27,

\[
C_1 = \frac{k\varepsilon_0 A}{d} = 6.64 \times 10^{-11} \text{ F}.
\]

This leads to

\[
q_1 = C_1 V_1 = 8.00 \times 10^{-10} \text{ C}, \quad q_2 = C_2 V_2 = 2.66 \times 10^{-10} \text{ C}.
\]

The addition of these gives the desired result: \( q_{\text{tot}} = 1.06 \times 10^{-9} \text{ C} \). Alternatively, the circuit could be reduced to find the \( q_{\text{tot}} \).

47. The capacitance is given by

\[
C = \kappa C_0 = \kappa\varepsilon_0 A/d,
\]
where \( C_0 \) is the capacitance without the dielectric, \( \kappa \) is the dielectric constant, \( A \) is the plate area, and \( d \) is the plate separation. The electric field between the plates is given by \( E = V/d \), where \( V \) is the potential difference between the plates. Thus, \( d = V/E \) and \( C = \kappa\varepsilon_0 AE/V \). Thus,

\[
A = \frac{CV}{\kappa\varepsilon_0 E}.
\]

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

\[
A = \frac{(7.0 \times 10^{-8} \text{ F})(4.0 \times 10^3 \text{ V})}{2.8(8.85 \times 10^{-12} \text{ F/m})(18 \times 10^6 \text{ V/m})} = 0.63 \text{ m}^2.
\]

48. The capacitor can be viewed as two capacitors \( C_1 \) and \( C_2 \) in parallel, each with surface area \( A/2 \) and plate separation \( d \), filled with dielectric materials with dielectric constants \( \kappa_1 \) and \( \kappa_2 \), respectively. Thus, (in SI units),

\[
C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1 + \kappa_2}{2} \right)
\]

\[
= \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.56 \times 10^{-4} \text{ m}^2)}{5.56 \times 10^{-3} \text{ m}} \left( \frac{7.00 + 12.00}{2} \right) = 8.41 \times 10^{-12} \text{ F}.
\]

49. We assume there is charge \( q \) on one plate and charge \( -q \) on the other. The electric field in the lower half of the region between the plates is

\[
E_1 = \frac{q}{\kappa_1\varepsilon_0 A}.
\]
where $A$ is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A}.$$ 

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2 \varepsilon_0 A} \left[ \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right] = \frac{qd}{2 \varepsilon_0 A} \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2 \varepsilon_0 A \kappa_1 \kappa_2}{d \kappa_1 + \kappa_2}.$$ 

This expression is exactly the same as that for $C_{eq}$ of two capacitors in series, one with dielectric constant $\kappa_1$ and the other with dielectric constant $\kappa_2$. Each has plate area $A$ and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area $A$, plate separation $d$, and dielectric constant $\kappa_1$.

With $A=7.89 \times 10^{-4} \text{m}^2$, $d=4.62 \times 10^{-3} \text{m}$, $\kappa_1 = 11.0$, and $\kappa_2 = 12.0$, the capacitance is

$$C = \frac{2(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)(7.89 \times 10^{-4} \text{m}^2)(11.0)(12.0)}{4.62 \times 10^{-3} \text{m} \cdot 11.0 + 12.0} = 1.73 \times 10^{-11} \text{F}.$$

50. Let

$$C_1 = \varepsilon_0 (A/2) \kappa_1 / 2d = \varepsilon_0 A \kappa_1 / 4d,$$

$$C_2 = \varepsilon_0 (A/2) \kappa_2 / d = \varepsilon_0 A \kappa_2 / 2d,$$

$$C_3 = \varepsilon_0 A \kappa_3 / 2d.$$

Note that $C_2$ and $C_3$ are effectively connected in series, while $C_1$ is effectively connected in parallel with the $C_2$-$C_3$ combination. Thus,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \varepsilon_0 A \frac{\kappa_1}{4d} + \frac{(\varepsilon_0 A / d) (\kappa_2 / 2) (\kappa_3 / 2)}{\kappa_2 / 2 + \kappa_3 / 2} = \varepsilon_0 \frac{A}{4d} \left( \kappa_1 + \frac{2\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right).$$

With $A=1.05 \times 10^{-3} \text{m}^2$, $d=3.56 \times 10^{-3} \text{m}$, $\kappa_1 = 21.0$, $\kappa_2 = 42.0$ and $\kappa_3 = 58.0$, we find the capacitance to be

$$C = \frac{(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)(1.05 \times 10^{-3} \text{m}^2)(21.0 + \frac{2(42.0)(58.0)}{42.0 + 58.0})}{4(3.56 \times 10^{-3} \text{m})} = 4.55 \times 10^{-11} \text{F}.$$