Friday, September 19, 2003

Classical Mechanics and Statistical Mechanics

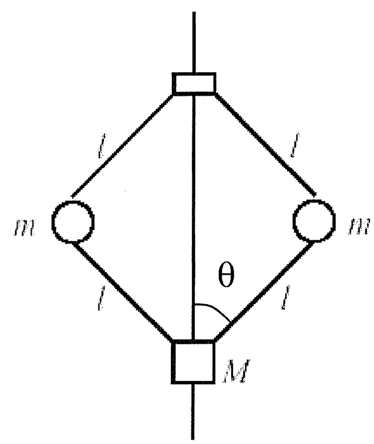
This examination has two parts, Classical and Statistical Mechanics. You need to complete two out of three problems on both sections of the exam for a total of four problems.

Classical Mechanics - choose 2 out of 3

1. A mechanical governor consists of two weights of mass m attached by massless rigid rods to two hinged pivots. The upper pivot is massless and cannot move in the vertical direction. The lower pivot has mass M and is free to slide up and down the center pole. All pivots and hinges are frictionless. The entire apparatus is forced to rotate at a fixed angular velocity ω about the vertical center pole. Gravity acts in the downward direction. Let g denote the acceleration of gravity.

   (a) Find the equilibrium value of $\cos\theta$.

   (b) Find the equation of motion for the angle θ and use it to determine the frequency of small amplitude oscillations about equilibrium.



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2. A particle executes elliptical motion about a force center. The central force is k/r^2.

   (a) Express the major and minor axes of the elliptical orbit in terms of the total energy E and angular momentum l.

   (b) Assume the initial orbit is nearly circular. At some point in the orbit a tangential impulse is applied to the particle, changing the velocity from v to $v + \delta v$. The particle remains elliptical motion after the velocity change. Show that the resulting relative change in both the major and minor axes of the orbit is twice the relative change in the velocity and that the axes are increased if $\delta v > 0$.

3. A homogenous cube of mass M, each edge of which has a length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall.

   (a) Determine the angular velocity of the cube when one face strikes the plane if the edge cannot slide on the plane.

   (b) Determine the angular velocity of the cube when one face strikes the plane if sliding can occur without friction.

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Statistical Mechanics - choose 2 out of 3

1. Consider the nuclei of atoms in a crystalline solid to each have a spin of unity. Each nucleus can be in one of three quantum states, described by quantum number m corresponding to $m = -1, 0, +1$. A nucleus has the same energy $E = \epsilon$ in the states $m = -1$ and $+1$ compared to an energy $E = 0$ in the state $m = 0$.

   (a) Find an expression, as a function of the absolute temperature, T, of the nuclear contribution of N nuclei to the internal energy of the solid.

   (b) Find an expression, as a function of T, of the nuclear contribution of N nuclei to the entropy of the solid. What is the behavior as T approaches zero and T approaches ∞.

   (c) Find an expression, as a function of T, of the nuclear contribution of N nuclei to the heat capacity of the solid. What is its temperature dependence for large values of T.

2. An ideal monatomic gas of N particles, each of mass m, is in thermal equilibrium at absolute temperature T. The gas is contained in a cubical box of side L, whose top and bottom sides are parallel to the earth's surface. The effect of the earth's uniform gravitational field on the particles should be considered, the acceleration due to gravity being g.

   (a) What is the average kinetic energy of a particle?

   (b) What is the average potential energy of a particle?

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3. A magnetic material in an applied magnetic field, H, has the fundamental thermodynamic relation

$$dE = TdS - PdV - MdH$$

where E, S, P, V, and M are the internal energy, entropy, pressure, volume, and magnetization respectively. You may assume that V is independent of H.

   (a) Find the Maxwell relation for $\frac{\partial S}{\partial H}|_T$, assuming that the volume doesn't change: $dV = 0$.

   (b) Calculate how the temperature changes with adiabatic changes in the magnetic field. Write your answer in terms of M, T, and the heat capacity at constant magnetic field, $C_H = T\left(\frac{\partial S}{\partial T}\right)|_H$.

   (c) If $M = \alpha VH/T$ and the coefficient $\alpha > 0$, determine if the temperature increases or decreases as the magnetic field is reduced.