Wednesday, September 17, 2003

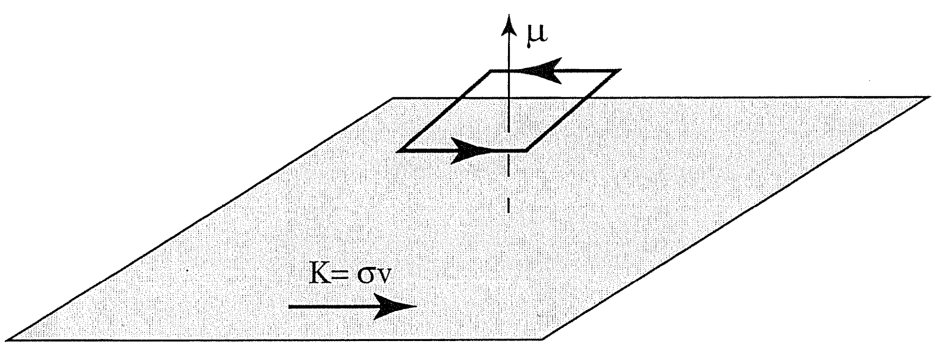
# Electricity and Magnetism

**Choose 3 out of 4 problems**

1. A stationary magnetic dipole, $\vec{\mu}$ , is situated above an infinite uniform surface current $\overrightarrow{K}$ .

   (a) Find the torque on the dipole.

   (b) Suppose that the surface current consists of a uniform surface charge $\sigma$ , moving at velocity $\overrightarrow{v} = v\hat{x}$ , so that $\overrightarrow{K} = \sigma\overrightarrow{v}$ , and the magnetic dipole consists of a uniform line charge $\lambda$ , circulating at speed $v$ around a square loop of side $l$, so that $\mu = vl^2$. Examine the same configuration from the point of view of system $S$, moving in the $x$ direction at speed $v$. In $S$ the surface charge is at rest, so it generates no magnetic field. Show that in this frame the current loop carries an electric dipole moment, and calculate the resulting torque.




Reminder: If two frames $F$ and $F'$ are moving relative to each other with relative velocity $\vec{\mathbf{V}}$, the components of a four-vector transform like: $X'_t = \gamma(X_t - \vec{\beta} \cdot \vec{\mathbf{X}})$; $X'_\parallel = \gamma(X_\parallel - \beta X_t)$; $\vec{\mathbf{X}}'_\perp = \vec{\mathbf{X}}_\perp$
$\gamma = 1/\sqrt{(1-\beta^2)}$; $\vec{\beta} = \vec{\mathbf{V}}/c$

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2. In a perfect conductor, the conductivity is infinite. A superconductor is a perfect conductor with the additional property that the magnetic field inside is zero.

   (a) Show that the magnetic field is constant in time inside a perfect conductor.

   (b) Show that the magnetic flux through a perfectly conducting loop is constant.

   (c) Show that the current in a superconductor is confined to the surface.

   (d) Superconductivity is lost above a certain critical temperature ($T_c$), which varies from one material to another. Suppose you have a sphere (radius $a$) above its critical temperature, and you hold it in a uniform magnetic field $\overrightarrow{B} = B_0\hat{z}$ while cooling it below $T_c$. Find the induced surface current density as function of polar angle $\theta$.

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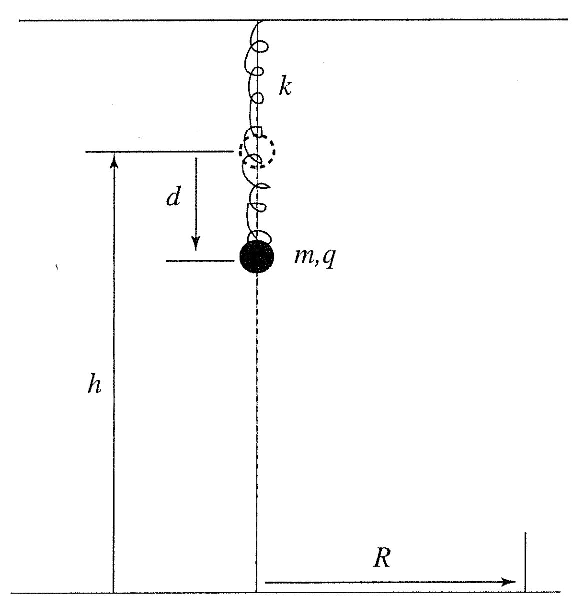
3. A particle of mass $m$ and charge $q$ is attached to a spring with force constant $k$, hanging from the ceiling. Its equilibrium position is a distance $h$ above the floor. It is pulled down in a distance $d$ below equilibrium and released at time $t = 0$.

(a) Under the assumption that $d \ll \lambda = \frac{2\pi c}{\omega} \ll h$, where $\omega$ is the oscillation frequency, calculate the intensity of the radiation hitting the floor, as a function of the distance $R$ from the point directly below $q$. Note the intensity here is the average power per unit area of the floor.

(b) At what $R$ is the radiation most intense?

(c) Because it is losing energy in the form of radiation, the amplitude of the oscillation will gradually decrease. Assume the fraction of the total energy lost in one cycle is very small. After what time $\tau$ has the amplitude been reduced to $d/e$.

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4. A familiar demonstration of superconductivity is the levitation of a magnet over a piece of superconducting material. Assume that the superconductor occupies the entire half-space below the $xy$ plane. Treat the magnet as a perfect dipole $\mu$ , a height $z$ above the origin, and constrained to point in the $z$ direction. The mass of the magnet is $M$.

(a) Find the force applied by the superconductor on the magnet.

(b) Determine the height at which the magnet will float.