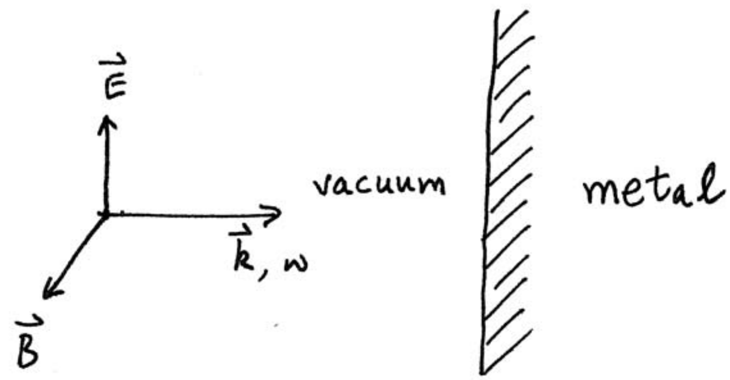
Department of Physics & Astronomy Qualifying Exam
Electricity and Magnetism

Problem 1



A plane electromagnetic wave of frequency ω and wave vector $\vec{k}$ is normally incident on a metal surface. The constitutive relations in the metal are

$$\vec{j}_\omega = \sigma(\omega)\,\vec{E}_\omega\,,\ \vec{D} = \vec{E}\,,\ \vec{B} = \vec{H}$$
$$\vec{D} = \varepsilon_0\vec{E}\,,\ \vec{B} = \mu_0\vec{H}\ \text{(SI)}$$

Here,

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}\ ;\ \sigma_0 = \begin{cases} ne^2\tau/m & \text{(Gaussian)} \\ ne^2\tau/4\pi\varepsilon_0 m & \text{(SI)} \end{cases}$$

and $\vec{j}_\omega$ and $\vec{E}_\omega$ are the current and electric field in the frequency domain:

$$(X_\omega = \int_{-\infty}^{\infty} X(t)e^{i\omega t}dt\ \text{ for any quantity X})$$

Further
$$\omega_p^2 \equiv \begin{cases} 4\pi ne^2/m & \text{(Gaussian)} \\ ne^2/m\varepsilon_0 & \text{(SI)} \end{cases}$$

and $\omega_p \gg 1/\tau$.

(a) Find the fraction of incident intensity reflected by the metal for $\omega \gg \omega_p$.
(b) Repeat part (a) for $\omega = \omega_p$.

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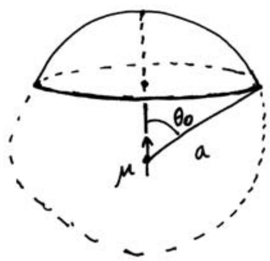
Problem 2

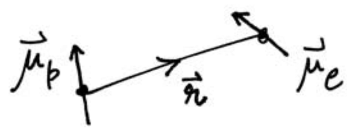
(a) For a point magnetic dipole, $\vec{\mu}$, located at the origin, the vector potential is given by

$$\vec{A} = \left[\frac{\mu_0}{4\pi}\right]\frac{\vec{\mu}\times\vec{r}}{r^3}$$

(The factor $\mu_0/4\pi$ is present only in the SI system.)

Show that this is correct by taking $\vec{\mu}\,||\,\hat{z}$, calculating the magnetic flux due to a dipole through a spherical cap of radius a, and half-angle θ_0 as shown, and equating this flux to the circulation of A around the edge of this cap.





(b) In the hydrogen atom, in addition to the Coulomb interaction energy there is an interaction between the magnetic dipole moments of the proton and the electron, $\vec{\mu}_p$ and $\vec{\mu}_e$. Show that in addition to the usual long-ranged part (that decays as $1/r^3$), the dipole – dipole interaction also contains a part V, where,

$$V(\vec{r}) \equiv K\vec{\mu}_e \bullet \vec{\mu}_p \delta^{(3)}(\vec{r}).$$

Find K.

(c) In the ground state of H, the electron wave function is given by

$$\Psi_e(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad (a_0 = \text{Bohr radius})$$

Calculate $\langle V\rangle = \int V(\vec{r})\,|\Psi_e(\vec{r})|^2 d^3r$, and find the difference in energy (i.e. in $\langle V\rangle$) between the configurations where $\vec{\mu}_e$ and $\vec{\mu}_p$ are parallel, and antiparallel.

<u>Useful identities:</u>

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\bullet\vec{C})\vec{B} - (\vec{A}\bullet\vec{B})\vec{C}$$

$$\nabla^2\frac{1}{r} = -4\pi\delta^{(3)}(\vec{r})$$

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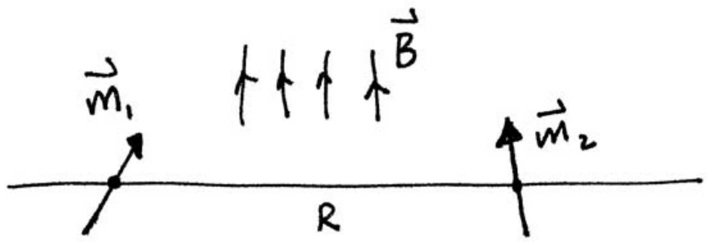
Problem 3

Two small magnetic dipoles $\vec{m}_1$ & $\vec{m}_2$ of equal magnitude m have their centers fixed, but can turn about them in a uniform magnetic field B which is perpendicular to the line joining their centers.



(a) Show that if B is very large, the lowest energy configuration is that in which both $\vec{m}_1$ and $\vec{m}_2$ point along $\vec{B}$.

(b) Show that the configuration of part (a) is stable as long as B is greater than some value B_0. Find B_0.

(Caution: The problem only asks you to think about stability. Do not try to find the lowest energy configuration for general B – that is a difficult problem.)

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Problem 4

Consider a nonmagnetic conducting spherical artificial satellite of radius *a* moving in an equatorial orbit with a constant velocity $\upsilon \ll c$ (speed of light). The space around the satellite may be considered to be nonconducting (vacuum).

(a) As a model problem, consider the satellite as a static conducting sphere of radius *a* that is placed in a uniform external electric field **E**. Find how the presence of the sphere alters the field.

(b) Now consider a better model, in which the motion of the satellite and the earth's magnetic field, **B**, are included, but there is no applied external electric field **E.** Determine the induced electric field **E*** around the satellite, and show the satellite acquires induced surface charges which give it an electric dipole of moment *p*. Find *p*.