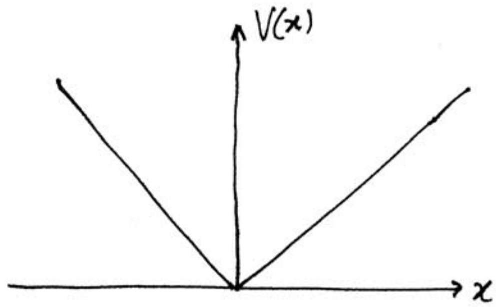
Department of Physics & Astronomy Qualifying Exam
Quantum Mechanics

Problem 1

Consider a particle moving in the one-dimensional potential

$$V(x) = \alpha |x|$$



(a) Estimate the ground state energy using a one-parameter variational wave function.

$$\psi(x) = (\text{const.})\, e^{-q|x/2|}$$

(b) Find all energy eigenvalues using the Bohr-Sommerfeld quantization rule.

(c) Make a variational estimate of the energy of the first excited state, by choosing a suitable trial wave function.

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Problem 2

A one-dimensional harmonic oscillator has mass *m* and angular frequency *ω*. A time-dependent state $|\psi(t)\rangle$ is given at *t*=0 by:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2s}} \sum |n\rangle$$

where |*n*> is an eigenstate corresponding to the quantum number *n*, and the summation runs from *N*-*s* to *N*+*s*, where *N*>>*s*>>1.

(a) What is <*E*>?
(b) Calculate the expectation value of x quantum mechanically. Show that it varies sinusoidally; find the frequency and amplitude.
(c) Compare the amplitude and frequency to the corresponding values for a classical oscillator.

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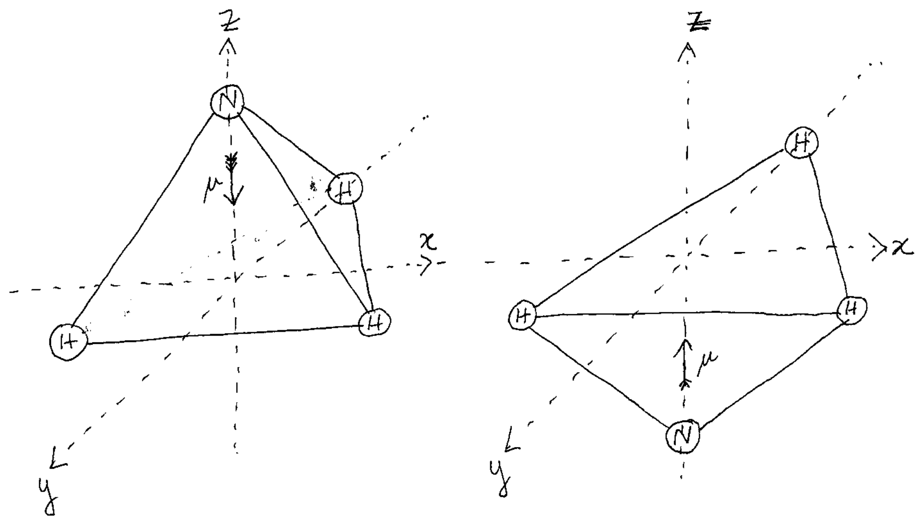
Problem 3

In the ammonia (NH_3) molecule, the nitrogen atom can be either above or below the plane of the three hydrogen atoms. These two states (shown below) are degenerate in the absence of external fields:



The solid lines are guides to the eye, not necessarily valence bonds. The triangle formed by the three H atoms is equilateral and in the x-y plane; the N atom is centered above or below the triangle.

We will write these states as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Even when there are no external fields, these states are found to be time-dependent, which means that the Hamiltonian, H, is not diagonal in this representation. By symmetry the diagonal elements are equal, and the non-diagonal elements are also equal, thus we can say that H = $\begin{pmatrix} E_o & \Delta \\ \Delta & E_o \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of this Hamiltonian. Discuss briefly what the eigenstates represent.

(b) The molecule has an electric dipole moment μ, pointing from the nitrogen atom towards the x-y plane of the hydrogen atoms. Write a modified Hamiltonian in the presence of an electric field **E** in the z-direction, and find the eigenvalues.

(c) Write the eigenvalues in the small-Δ limit. What is the permanent dipole moment of each eigenstate?

(d) Write the eigenvalues in the small **E** limit, to lowest significant order in **E**. Also write the eigenstates to zeroth order in **E**. What is the dipole moment of each eigenstate?

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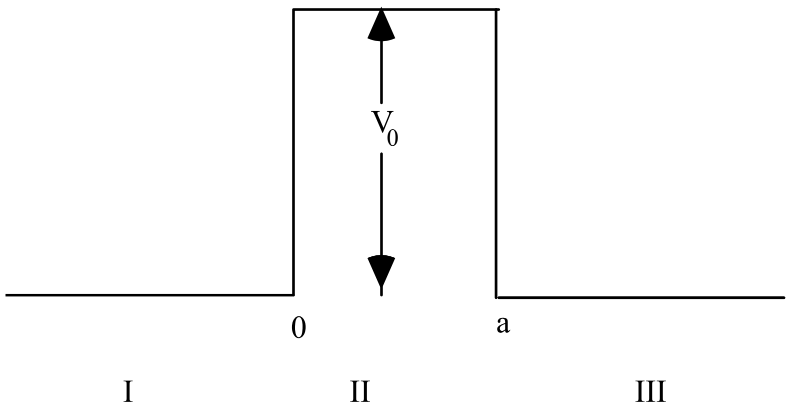
Problem 4

a) Calculate the transmission coefficient through the step barrier shown below in the limit where the potential, V_0, is large relative to the kinetic energy of the incoming wave, and the overall transmission coefficient is small.



b) Assume initially that a particle is localized in region I of the system drawn below and that the potential is very large (so that the wave function in region II essentially vanishes). Find the wave function of the lowest bound state (which is a standing wave). Decompose this standing wave into two traveling waves. Now assume the transmission coefficient from part (a) is finite but small, and calculate the probability flux through the barriers. Since this will lead to a decay of the probability of the particle being in region I, calculate the rate of decay of the initial state. This simple problem is a model for what is called a virtual state in quantum mechanics.