Physics Qualifier Problems

Classical Mechanics and Statistical Mechanics

September 15, 2004

Instructions:

Problems 1–3 pertain to classical mechanics, while problems 4–6 pertain to statistical mechanics.

- Do **2** of the problems 1, 2 and 3.
- Do **2** of the problems 4, 5 and 6.

Thus, you will do four problems in total – two in classical mechanics, and two in statistical mechanics.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code.*

Write all your answers in blue books. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

Good Luck!

1. An automobile is started from rest with one of its doors initially at right angles to the body of the car. If the frictionless hinges of the door are toward the front of the car, the door will slam shut as the automobile picks up speed. The car starts moving forward with constant acceleration, a, the radius of gyration of the door about the axis of the hinges is r_0, and the center of mass of the door is a distance r from the hinges.

   (a) Derive the equations of motion for the position of the center of mass of the door, in terms of the forces acting on the door.

   (b) Derive the equation for the rotation of the door about its center of mass in terms of the torques acting on the door.

   (c) Obtain (from (a) and (b) or otherwise) the equation for the angular acceleration, $d^2\theta/dt^2$, of the door.

   (d) Calculate the angular speed $d\theta/dt$ of the door.

   (e) Finally, obtain a formula for the time needed for the door to close.

2. A particle of mass m is constrained to slide under gravity along a smooth straight wire, one end of which lies at the origin of a Cartesian coordinate system. The wire makes an angle, α with the $z-$axis and rotates about this axis with a constant angular velocity ω. Let $r(t)$ denote the distance of the particle from the origin at time t, and assume that the gravitational acceleration points in the negative z direction.

   (a) Determine the Lagrangian and the equation of motion of this system.

   (b) Suppose that initially the particle is at rest in the rotating system, and at a distance $r(0) = r_0$ from the origin. Determine the equilibrium point and describe quantitatively the motion of the particle.

3. Two equal point masses m move in one dimension at the junctions of three springs, as shown in the figure below. The (massless) springs all have unstretched lengths a. The spring constants and rigid force constraints at both ends are shown in the figure. The local displacements of the masses are x_1 and x_2.

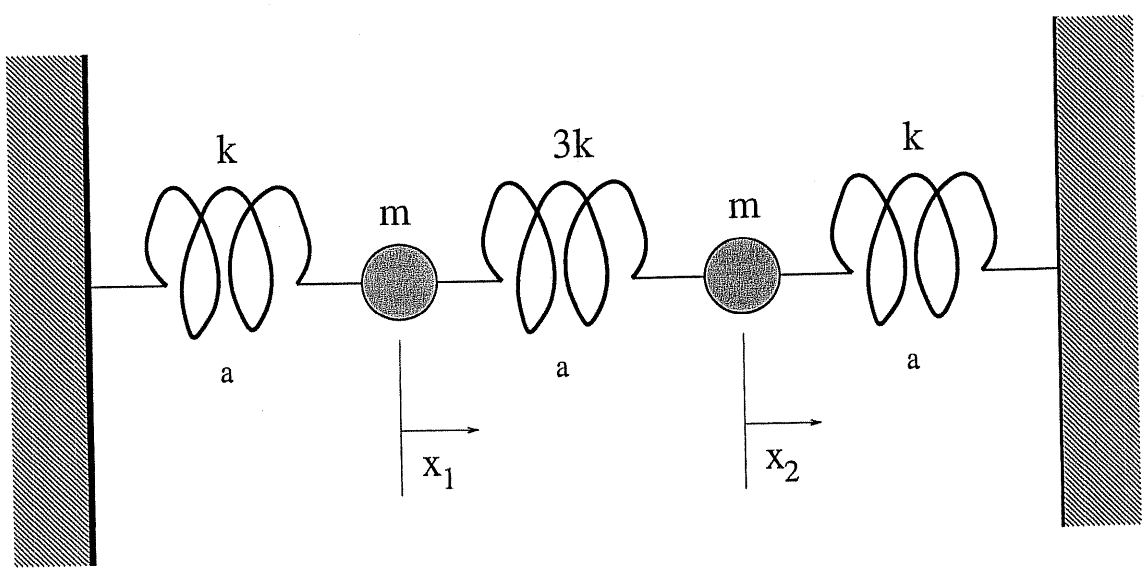
   (a) Find the total kinetic energy, the total potential energy, and the Lagrangian for the system.

   (b) Derive the equations of motion.

   (c) Find the characteristic frequencies and corresponding normal modes of motion.

   (d) If both constraints are removed, leaving both of the ends free, but with the system still aligned in one dimension, what is the Lagrangian and the equations of motion?

   (e) Prove, in this latter case, that there is a normal mode with zero frequency, and describe the motion corresponding to this zero-frequency mode.



4. Einstein devised a simple model for explaining the thermal properties of a solid. This "Einstein solid" consists of a collection of N independent, quantum mechanical, one-dimensional harmonic oscillators, each with the same frequency, ν.

   (a) Using this model, derive an expression for the heat capacity as a function of temperature.

   (b) One can also approach this problem using classical physics. Use the Equipartition Theorem to derive the classical expression for the heat capacity of a collection of N independent one-dimensional harmonic oscillators.

   (c) Show that your answer to part (a) agrees with your answer to part (b) for very high temperatures, while it predicts that the heat capacity vanishes at low temperatures.

5. Consider a system that is in thermal and diffusive equilibrium with a reservoir. Show that the average number of particles in the system is given by:

$$N_{avg} = \frac{kT}{Z_G}\frac{\partial Z_G}{\partial \mu}$$

   where Z_G is the grand partition function, μ is the chemical potential, and the partial derivative is taken at a fixed temperature and volume.

6. For an electron whose total energy is much larger than its rest mass energy, the relation between energy, E, and momentum, p, can be approximated by the extreme relativistic expression $E = pc$. This approximation can be used for an electron gas if the electron density is so high that the Fermi energy is much greater than the rest mass energy.

   (a) For a highly relativistic electron gas occupying a fixed volume, V, derive expressions for the density of states and the Fermi energy. For the latter, express your answer in terms of the electron density n.

   (b) For a fully degenerate electron gas, find an approximate expression for the number density corresponding to the transition between the non-relativistic and relativistic regimes.