Physics Qualifier Problems
Electricity and Magnetism

September 16, 2004

Instructions:

Four problems are stated on these pages.

Solve **3** of these problems.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code*.

Write all your answers in blue books. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

Good Luck!

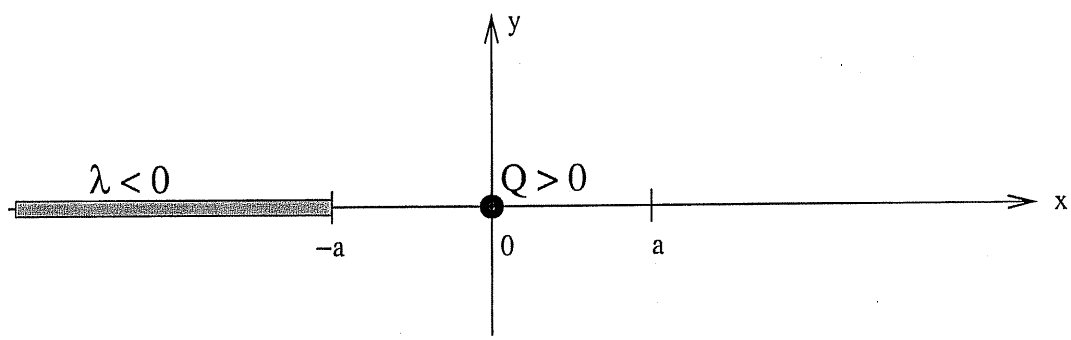
1. Consider a single point charge $Q > 0$ placed at the origin. The negative x axis for $x < -a$ is covered by a thin charge distribution with linear charge density $\lambda < 0$, as shown in the figure. Both the point charge and the linear charge distribution are fixed in place.

   (a) What is the force on the charge Q (magnitude and direction)?

   (b) What is the potential at positive $x = a$ due to the charge Q and the line of charge? Define your potential such that $\Phi = 0$ at $x = a/2$.

   (c) Another positive test charge, $q > 0$, is placed on the x-axis at $x = a$. Consider small displacements $\delta = x - a$ with $|\delta| \ll a$ along the x-axis, find the value of Q for which the point $x = a$ is an equilibrium.

   (d) Consider displacements along the x-axis only. Is this equilibrium stable or unstable? Explain your reasoning.



2. Two long wires lie in the xz-plane, and they run parallel to the x-axis. They pass through $z = \pm a$, as shown in the figure. The wire on the left carries current I in the positive $\hat{x}$-direction, and supports a constant charge density $\lambda > 0$. The wire on the right carries current I in the opposite direction, and supports a negative charge density $-\lambda < 0$.

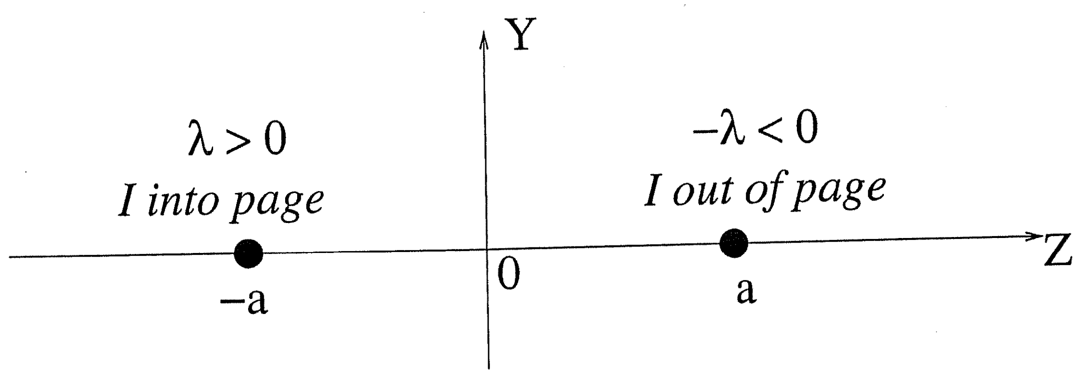
(a) Use Gauss's Law to derive the individual electric fields for the wires.

(b) Use Ampère's Law to derive the individual magnetic fields for the wires.

(c) What is the net force per unit length acting on the wires?

(d) Find a condition between λ and I which gives zero net force, if possible.

(e) Describe how you might construct this system using wires, batteries, and resistors.



(X−axis points into the page.)

3. An electromagnetic wave of frequency ω propagates in the positive x-direction in a non-conducting infinite medium with an index refraction $n(x)$ which varies slowly according to

$$n(x) = 1 + \epsilon \exp(-\frac{1}{2}\frac{x^2}{\sigma^2}) \tag{1}$$

where $\epsilon \ll 1$ and $\sigma \gg \lambda$.

(a) What is the local wavelength as a function of position, $\lambda(x)$?

(b) What is the velocity of propagation, $v(x)$?

(c) Find an approximate solution to the wave equation, taking the phase velocity to be approximately constant at a given position, x.

(d) What is the phase shift $\Delta\phi$ at $x \to \infty$ relative to the same wave propagating in vacuum?

Reminder:

The definite Gaussian integral is

$$\int_{-\infty}^{\infty} dx \, \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2}\frac{x^2}{\sigma^2}) = 1.$$

4. A point-like particle with charge Q sits stationary at the origin of a Cartesian coordinate system, (x, y, z).

   (a) What is the electric potential, ϕ, at an arbitrary point (x, y, z) due to this particle? Take the vector potential, $\vec{A}$, to vanish.

Consider a reference frame (x', y', z', t') in which the particle is moving at constant velocity $v < c$ along the x' axis. Recall that the scalar and vector potentials together constitute a 4-vector, $A_\mu = (\phi, \vec{A})$, and for this situation the rule for Lorentz transformations may be written

$$x' = \gamma(x + \beta ct) \qquad y' = y \qquad z' = z \qquad t' = \gamma(t + \beta x/c)$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$.

   (b) What are the scalar potential, ϕ', and vector potential $\vec{A}'$, in this reference frame? Express your answer in terms of the space and time coordinates of this reference frame, *i.e.*, (x', y', z', t').
   Verify that $A_\mu A^\mu \neq 0$ is invariant.

   (c) What is the electric field, $\vec{E}'$?
   Express your answer in terms of the vector $\vec{D} \equiv \gamma x'\hat{x}' + y'\hat{y}' + z'\hat{z}'$.

   (d) Since $\vec{E} \cdot \vec{B}$ is a Lorentz invariant, what qualitative statement can you make about the magnetic field, $\vec{B}'$? Compute the magnetic field explicitly, and describe it qualitatively. (It is useful to employ $\vec{v} = \beta c\hat{x}'$.)

   (e) In which direction does the Poynting vector, $\vec{S}'$, point?
   (*Hint:* consider the components of $\vec{S}'$ parallel and perpendicular to $\vec{v}$.)

   (f) What is the ratio, E'_y/E'_x, for large Lorentz boosts, $\gamma \gg 1$?

Reminder:

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$$