Electricity and Magnetism

Choose 3 out of 4 problems

1. A plane electromagnetic wave of angular frequency ω is incident normally on a slab of non-absorbing material. The surface lies in the $x - y$ plane. The material is anisotropic with $\epsilon_{xx} = n_x^2\epsilon_0$, $\epsilon_{yy} = n_y^2\epsilon_0$, $\epsilon_{zz} = n_z^2\epsilon_0$, $n_x \neq n_y \neq n_z$ and $\epsilon_{xy} = \epsilon_{yz} = \epsilon_{zx} = 0$. Assume that $\mu = \mu_0$ everywhere.

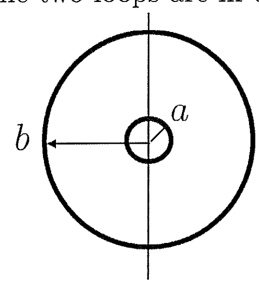
   (a) Write down the boundary conditions for $\vec{D}, \vec{E}, \vec{H}$ and $\vec{B}$ at the surface of an infinitely thick slab.

   (b) If the incident plane wave is linearly polarized with its electric field at 45^o to the x and the y axis, what will be the state of polarization of the reflected wave for an infinitely thick slab?

   (c) Now assume the slab has a thickness d, derive a set of conditions under which light with the E field along the y axis is completely reflected while light polarized with the E field along the x axis is maximally transmitted.

   (d) For the slab of arbitrary thickness d, derive an equation for the relative amplitude and phase of the transmitted electric field vectors for initial polarization along the x and y direction. (Part c is a special case of this.)

2. A small circular loop (radius a) lies in the center of a big circular loop (radius b). $a << b$. The two loops are in the same plane.

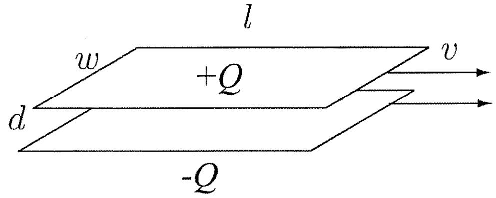


(a) A clockwise current $I(t)$ in the big loop is gradually increasing at a constant rate k. Find the emf induced in the small loop.

(b) If the current $I(t)$ flows in the small loop instead of the big loop, what is the emf induced in the big loop?

(c) Which way will the induced current flow in the big loop?

3. A parallel-plate capacitor moves at speed v in the lab frame. v is not much smaller than c. In the lab frame, the distance between the two plates is d and the width and length of the plates are w and l. $d << l, w$. The total charge on the top (or bottom) plate is $+Q$ (or $-Q$).



(a) In the laboratory frame, calculate the electromagnetic energy stored in the space between the plates.

(b) In the laboratory frame, calculate the electromagnetic momentum in the space between the plates.

Reminder: $\beta = v/c$; $\gamma = 1/\sqrt{1-\beta^2}$; $E' = \gamma(E - \beta p_{\|})$; $p'_{\|} = \gamma(p_{\|} - \beta E)$

4. An infinitely thin spherical shell of radius R and surface charge density σ is rotated with angular velocity ω about a diameter.

   (a) Determine the magnetostatic potential for the region inside and outside the shell. What are the boundary conditions required?

   (b) Find the magnetic field inside and outside the shell.