Physics Qualifier Problems
Classical Mechanics and Statistical Mechanics

September 14, 2005

Instructions:

Problems 1–3 pertain to classical mechanics, while problems 4–6 pertain to statistical mechanics.

- Do **2** of the problems 1, 2 and 3.
- Do **2** of the problems 4, 5 and 6.

Thus, you will do four problems in total – two in classical mechanics, and two in statistical mechanics.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code*.

Write your answers for each problem in a separate blue book. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

Good Luck!

(1) A uniform thin stick of length L and mass M leans against a frictionless wall. The angle between the stick and the floor is θ, as depicted below. Between the bottom of the stick and the floor, the coefficient of static friction is μ_s, while the coefficient of kinetic friction is μ_k.

(a) For a given value of θ, what is the smallest value of μ_s for which the stick will not slide?

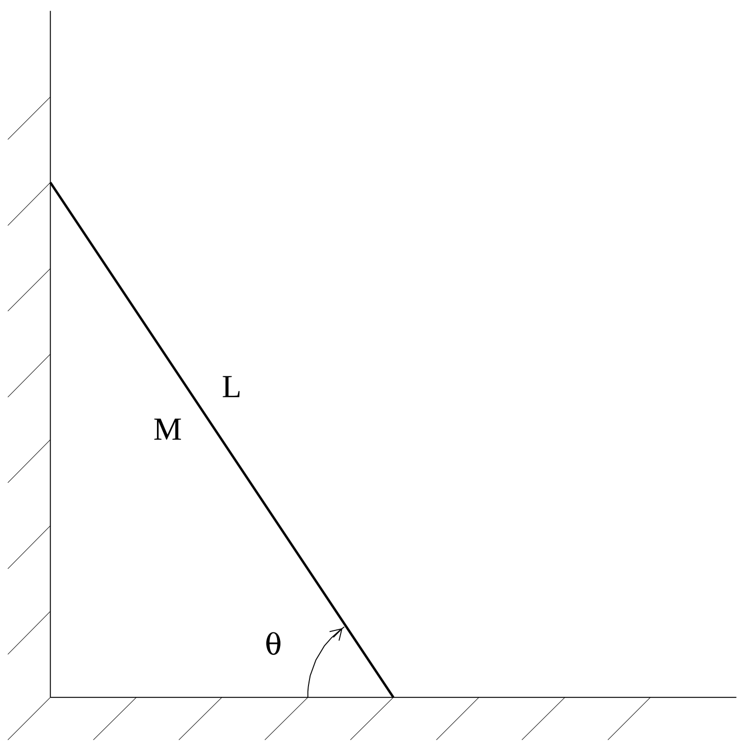
(b) Derive the equation of motion for $\theta(t)$.

(c) If the stick begins to slide at $t = 0$ ($\dot{\theta}(0) = 0$) from an angle $\theta(0) = \theta_0 < \pi/2$, what is the initial angular acceleration $\ddot{\theta}(0)$?

(d) Neglecting friction ($\mu_k = 0$), calculate the angular speed $\dot{\theta}(t)$, and obtain an expression for the total time it takes for the stick to fall.

(e) Neglecting friction ($\mu_k = 0$), find an expression for the stick's the potential and kinetic energies. Show that the total energy is conserved.

(f) Neglecting friction ($\mu_k = 0$), calculate the angular momentum of the stick about its center of mass. Is it conserved? Why, or why not?



(2) Three equal point masses of mass m have equilibrium positions at the vertices of an equilateral triangle. They are connected by equal springs with spring constant k that lie along the arcs of a circle circumscribing the triangle (see Figure below). The masses and the springs are constrained to move only on the circle.

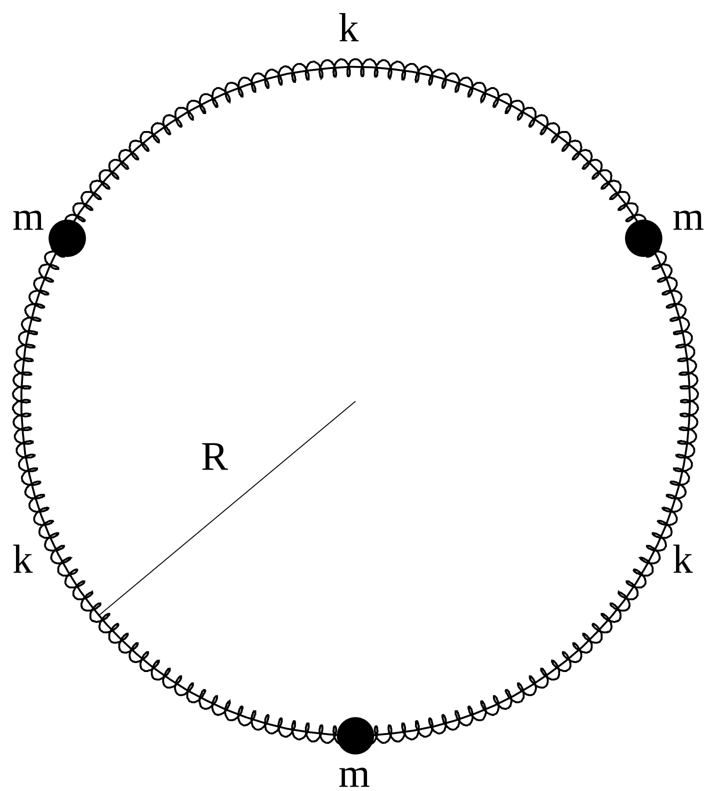
(a) Write the Lagrangian for the system.

(b) Obtain the equations of motion.

(c) Find the eigenfrequencies of oscillations.

(d) Find the corresponding eigenmodes, and qualitatively describe each normal mode of oscillation.

(e) If one of the springs has its force constant changed by an amount $\delta k \ll k$, what are the changes to the eigenfrequencies and eigenmodes, to first order in δk?



(3) Consider an infinitely long continuous string in which the tension is given as τ. A mass M is attached to the string at a position corresponding to $x = 0$. A wave train is incident from the left with velocity ω/k where ω is its frequency and k its wavenumber.

(a) What is the equation describing the propagation of the wave and the boundary conditions at $x = 0$?

(b) Find the reflection coefficient, R, and the transmission coefficient, T, at $x = 0$ in terms of τ, M, ω, and k.

(c) Determine the phase changes for the reflected and transmitted waves at $x = 0$.

(4) Consider a system of $N \gg 1$ non-interacting particles in which the energy of each particle can assume two distinct values: 0 and E ($E > 0$). Denote by n_0 and n_1 the occupation numbers of the energy levels 0 and E, respectively. The fixed total energy of the system is U.

(a) Calculate the entropy of the system.

(b) Find the temperature T, as a function of U. For what range of values of n_0 is $T < 0$?

(c) In which direction does heat flow when a system of negative temperature is brought into thermal contact with a system of positive temperature? Why?

(5) The following describes a method used to measure the adiabatic exponent γ of a gas. (We remind you that, for an ideal gas, PV^γ is constant if the gas evolves adiabatically)

The gas, assumed ideal, is confined within a vertical cylindrical container and supports a freely moving piston of mass m. The piston and cylinder both have the same cross-sectional area A. The atmospheric pressure is P_0, and when the piston is in equilibrium under the influence of gravity (acceleration g) and the gas pressure, the volume of the gas is V_0 . The piston is now displaced slightly from its equilibrium position and is found to oscillate about this position with frequency ν. The oscillations of the piston are slow enough that the gas always remains in internal equilibrium, but fast enough that the gas cannot exchange heat with the outside. The variations in gas pressure and volume are thus adiabatic. Express γ in terms of m, g, A, P_0 , V_0 , and ν .

(6) The figure below is a representation of the Einstein model of a solid. This model consists of N identical atoms (or molecules) of mass m, each of which is localized in each of the three orthogonal directions by restoring forces proportional to the corresponding displacements. In this model, the effects of one atom's displacement upon the neighboring atoms are ignored, and orthogonal oscillations of a given atom are considered to be independent.

For this problem, assume that the absolute temperature T is sufficiently high so that classical statistical mechanics is applicable.

(a) Find the heat capacity.

(b) Now imagine a modified Einstein model, in which each restoring force is proportional to the cube of the corresponding displacement. Find the heat capacity, again working in the high temperature limit.