Physics Qualifier Problems
Classical Mechanics and Statistical Mechanics

June 8, 2005

Instructions:

Problems 1–3 pertain to classical mechanics, while problems 4–6 pertain to statistical mechanics.

- Do **2** of the problems 1, 2 and 3.
- Do **2** of the problems 4, 5 and 6.

Thus, you will do four problems in total – two in classical mechanics, and two in statistical mechanics.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code*.

Write all your answers in blue books. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

Good Luck!

1. A thin uniform rod of mass, M, is initially supported at its ends as shown. At time, $t = 0$, the right hand support is removed, while the left end remains hinged to the bar without friction.

   a. Derive the equations of motion for the position of the center of mass of the rod in terms of the forces acting on the rod.

   b. Derive the equation of motion for the rotation of the rod about the hinged end and the center of mass.

   c. What force does the hinge exert on the bar immediately after the right hand support is removed, that is at $t = 0^+$?

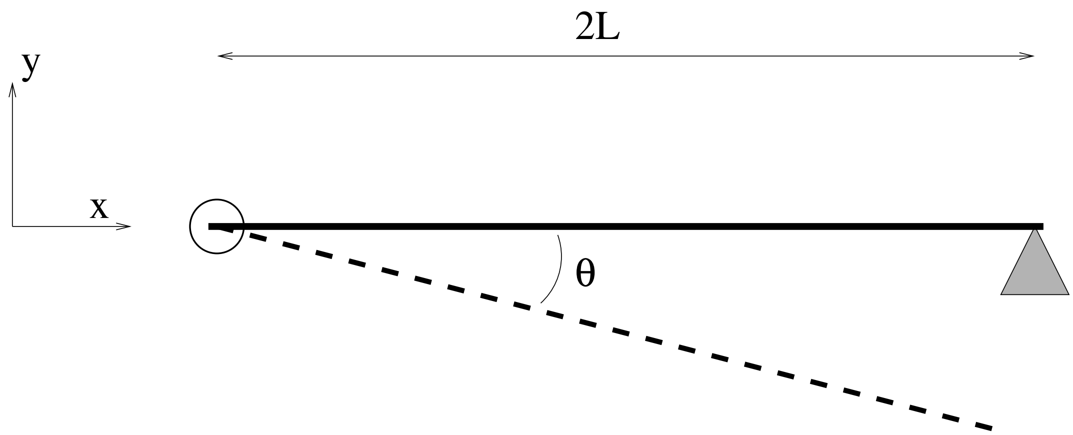
   d. Calculate the angular acceleration $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$ of the rod for $t > 0$.

   e. Calculate the angular speed $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ of the rod for $t > 0$.

   f. What is the kinetic energy at any angle θ?

   g. Calculate the potential energy, V, at any angle θ and demonstrate the total energy is conserved.

   h. At what time does θ reach its maximum value, $\theta = \pi$?

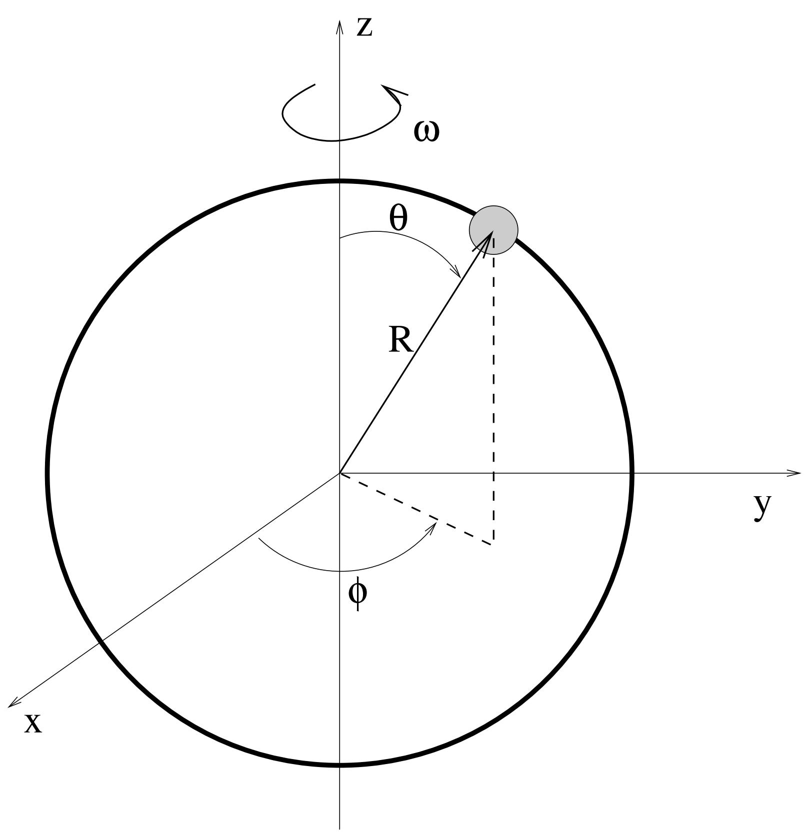


2. Consider a particle of mass, m, constrained to move (without friction) on a circular wire of radius, R. The wire is rotating with a constant angular velocity, $\omega = \dot{\phi}$, about a vertical diameter. The particle's motion is also subject to a gravitational acceleration, g. Gravity acts in the $-\hat{z}$ direction.

a. Derive the Lagrangian and the equation of motion for the particle.

b. Find the equilibrium position of the particle and calculate the frequency of small oscillations about this point.

c. Determine and interpret the critical frequency, ω_c, which distinguishes the different types of particle motion.



3. A point mass, m, describes a circular orbit, $r = 2R\cos\theta$, with $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, under the influence of an attractive central force, $F(r)$, directed toward a point P on the circle, as shown.

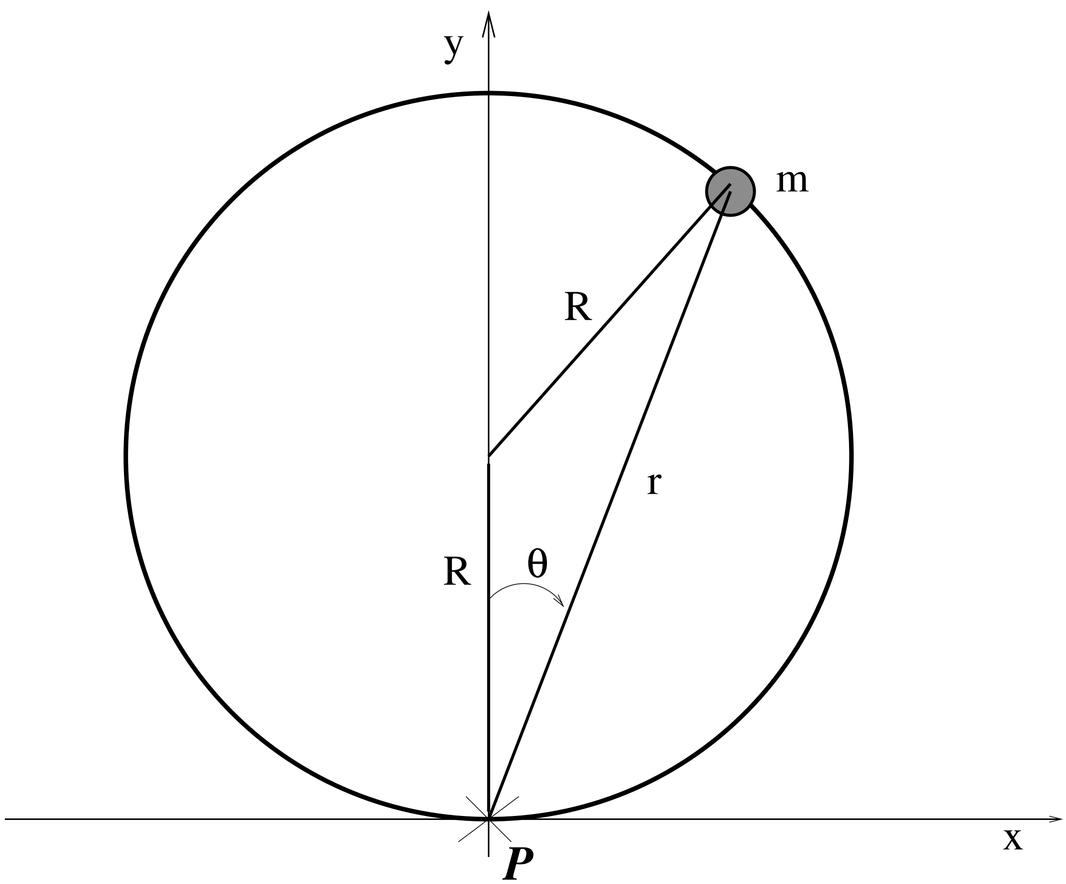
a. Prove the angular momentum and total energy are conserved (first integrals).

b. Using the orbit equation for $r(\theta)$ show the force, $F(r)$, varies as the inverse fifth power of the distance.

c. Show the total energy is zero.

d. Find the period of motion.

e. Calculate dx/dt, dy/dt, and the speed v around the orbit and determine how they behave as the particle passes through the force center.



4. In a gaseous environment such as the surface of the Sun, where Hydrogen can be found in both its ionized and neutral state, the relative population in the two states for thermal equilibrium is given by the following equation (originally derived by Saha):

$$\frac{n_p}{n_H} = \left(\frac{2\pi m_e\, kT}{h^2}\right)^{3/2} n_e^{-1}\, e^{-I/kT}$$

Here n_p, n_H, and n_e are respectively the number of protons, neutral Hydrogen atoms, and electrons per unit volume, I is the ionization potential of Hydrogen (13.6 eV), T is the temperature, m_e is the electron mass, and h and k are the Planck and Boltzmann constants.

Derive this equation. Assume that the temperature is low enough such that neutral Hydrogen atoms are all in their ground state. Further, assume that the density is low enough such that the ideal gas law can be used to treat the free electrons.

It may help to know that the entropy of a monatomic ideal gas consisting of N particles of mass m occupying volume V is given by the Sackur-Tetrode equation:

$$S = Nk\left(\frac{5}{2} + \ln\left[\frac{V(2\pi mkT)^{3/2}}{Nh^3}\right]\right)$$

5. A homogeneous substance at temperature T and pressure p has a molar volume v and a molar specific heat (measured at constant pressure) given by c_p. Its coefficient of volume expansion α is known as a function of temperature. Calculate how c_p depends on the pressure at a given temperature, *i.e.*, calculate $(\partial c_p/\partial p)_T$, expressing the result in terms of T, v, and the properties of α.

[The coefficient of volume expansion, α, is defined as the rate of change of volume with respect to temperature (at constant pressure) divided by the volume:

$$\alpha \equiv \left.\frac{d\ln V}{dT}\right|_p .$$

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6. (A) Use the First Law of Thermodynamics and the ideal gas law to show that for an ideal gas, an adiabatic process resulting in a change in volume will produce a change in pressure given by

$$\left(\frac{p_f}{p_i}\right) = \left(\frac{V_f}{V_i}\right)^{-\gamma}$$

where γ is the ratio of specific heats, c_p/c_v. (The subscripts i and f denote initial and final states.)

(B) A Carnot engine is an example of a device that does work by exchanging heat between itself and two heat reservoirs at different temperatures. The Carnot engine operates using a repeating cycle (the "Carnot cycle") that has four quasistatic steps:

(1) fluid, at temperature T_h, absorbs heat from the higher temperature bath while expanding.

(2) fluid continues to expand, but in thermal isolation, until fluid temperature is T_c.

(3) fluid, at temperature T_c, expels heat to the lower temperature bath while simultaneously being compressed.

(4) fluid continues to be compressed, in thermal isolation, until fluid temperature is T_h.

The temperatures of the baths are infinitesimally higher than T_h and infinitesimally lower than T_c, respectively.

The efficiency of a heat engine is the work done during one cycle, divided by the heat absorbed from the higher temperature bath during one cycle. For a Carnot engine that uses an ideal gas as a fluid, the efficiency equals $(1 - T_c/T_h)$. Derive this result by directly computing the ratio of heat absorbed to heat expelled. Do not use the Second Law of Thermodynamics.