Physics Qualifier Problems
Electricity and Magnetism

June 10, 2005

Instructions:

Four problems are stated on these pages.

Solve **3** of these problems.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code*.

Write all your answers in blue books. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

Good Luck!

1. (a) Write down Maxwell's Equations for a homogeneous medium of dielectric constant ϵ, for $\mu = 1$, and for conductivity σ.

   (b) Derive the wave equation for plane wave propagation ($\vec{E}$ and $\vec{B}$) and show that

$$k^2 = \frac{\epsilon}{c^2}\left(1 + i\frac{4\pi\sigma}{\omega\epsilon}\right).$$

   (c) Show that in the good conductor limit,

$$k \approx \frac{1}{\delta}(1+i)$$

   where the skin depth is $\delta = \sqrt{2\pi\omega\sigma}$.

2. There are two rings of radius R placed in the xy-plane, concentric with the z-axis. The first ring carries a static charge density

$$\rho = \frac{Q}{2\pi R} \cos(2\phi)$$

The second carries a steady current,

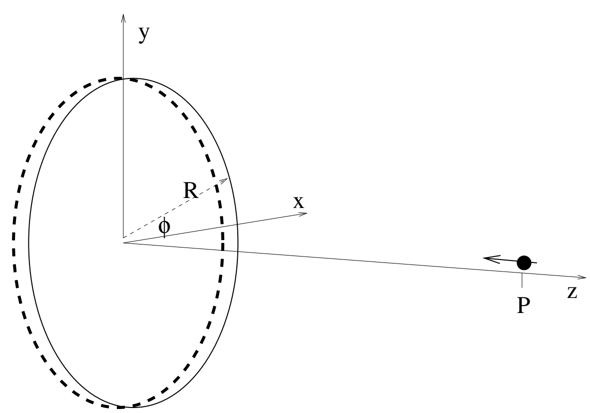
$$\vec{J} = I\,\hat{\phi}$$

You should neglect the thickness of the two rings, and the separation between them. The angle ϕ is measured from the x-axis.

(a) Find the multipole expansion for the scalar potential V for a point $\mathcal{P}$ at $z \gg R$ on the z-axis.

(b) Write an expression for the vector potential $\vec{A}$ at the same point, in terms of the magnetic dipole moment, $\vec{m}$. State explicitly what is $\vec{m}$ in terms of I and R.

(c) A positively charged particle ($q > 0$) travels parallel to the z-axis at point $\mathcal{P}$ with a small displacement $x \ll R \ll z$ in the x-direction. What approximate force is acting on the particle?



One ring holds a static electric charge distribution.
The other ring carries a steady current.
They are very close together but there is no electrical contact.

Reminder:

$$Q_{ij} = \int \left(3\,x'_i\,x'_j - r'^2\,\delta_{ij}\right)\,\rho(\vec{r}')\,d^3\vec{r}'$$

and

$$\int_0^{\pi} \cos^2 u\,du = \frac{\pi}{2}$$

3. Two infinite lines of constant linear charge density, λ, lie in the xy-plane at $y = \pm d$. The upper one has positive charge, $\lambda > 0$, and the second one has negative charge, $-\lambda < 0$. A positive charge Q of mass M is placed at rest on the z-axis at $z = L$.

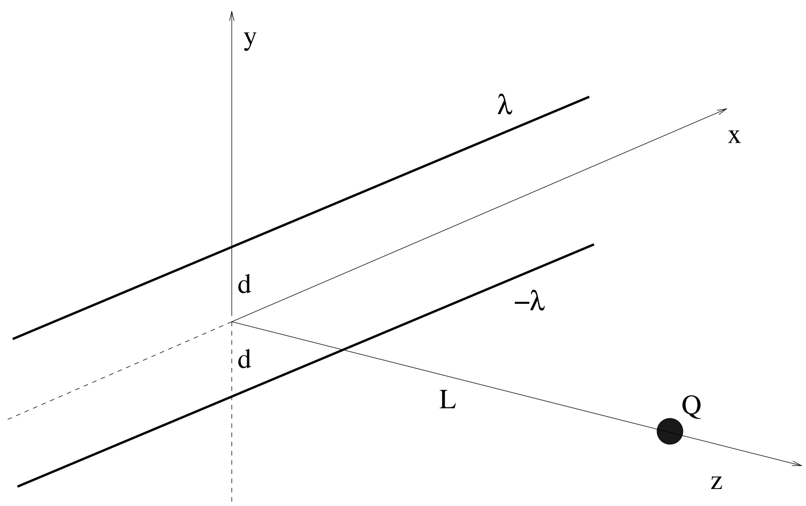
   Consider a "boost" of the lines of charge in the $+\hat{x}$-direction, with boost factor $\beta = v/c$.

   (a) What are the electric fields, $\vec{E}'$ and $\vec{B}'$, at a point along the z-axis, after this boost?

   (b) What is the force acting on the point charge, Q?

   (c) What is the trajectory of the point charge if it is released from rest at $t = 0$, for a small time t, in the lab frame?

   Please express your answers using d, L, λ, γ and β, etc. – not in terms of angles.



Reminder:
For a Lorentz boost in the x_1 direction,

$$
\begin{array}{ll}
E_1' = E_1 & B_1' = B_1 \\
E_2' = \gamma(E_2 - \beta B_3) & B_2' = \gamma(B_2 + \beta E_3) \\
E_3' = \gamma(E_3 + \beta B_2) & B_3' = \gamma(B_3 - \beta E_2)
\end{array}
$$

4. Two metal objects are embedded in a weakly conducting material of conductivity σ.

   (a) Show that the resistance between them is related to the capacitance of the arrangement by
   $$R = \frac{A}{\sigma C}$$
   and express A in terms of fundamental constants.

   (b) Connect a battery between the two metal objects and charge them up to a potential difference V_0. When you disconnect the battery, the charge will leak off the objects gradually. Show that
   $$V(t) = V_0\, e^{-t/\tau}$$
   and find the time constant τ in terms of ϵ_0 and σ.