# Physics Qualifier Problems
# Electricity and Magnetism

June 8, 2006

**Instructions:**

Four problems are stated on these pages.

Solve **3** of these problems.

Please indicate which problems you want graded for credit.

Do NOT write your name on the blue book – write your *code*.

Write your answers for each problem in a separate blue book. These sheets stating the problems will be collected and discarded at the end of the exam period.

You are allowed to use the single index card you prepared in advance.
You can refer to the mathematic reference book in the room.
You are NOT allowed to use notes or books, and calculators are not allowed.

*Good Luck!*

(1) $\eta_c$ **Production and Decay** – Consider the reaction $p + \bar{p} \to \eta_c$, where $p$ and $\bar{p}$ are the proton and the antiproton (both with equal and opposite charge and equal mass $m_p$) and $\eta_c$ is a pseudoscalar $c\bar{c}$ bound state of mass $m_\eta > 2m_p$. In the lab reference frame, the proton is at rest, while the antiproton has momentum $\vec{p}$.

(a) What is the smallest value of $|\vec{p}|$ required to produce the $\eta_c$?

(b) Under the conditions in (a), what is the speed $\beta$ of the antiproton and its boost factor $\gamma$ in the lab frame? In the lab frame, what are the threshold energy $E_\eta$ and momentum $\vec{q}$ of the $\eta_c$?

(c) One of the many decay modes of the $\eta_c$ is $\eta_c \to \gamma\gamma$, where $\gamma$ are photons. Since the $\eta_c$ is a scalar (no intrinsic angular momentum), in the reference frame where $\eta_c$ is at rest, the decay is isotropic. In this reference frame, what are the energies $E_{\gamma 1}$ and $E_{\gamma 2}$ of the two photons?

(d) In the laboratory reference frame (see (a) and (b)), what are the smallest and largest allowed values (respectively, $E_\gamma^{\text{min}}$ and $E_\gamma^{\text{max}}$) for the photon energies?

(e) Sketch the laboratory energy distribution of the photons.

(2) **Magnetic Moment in an External Magnetic Field**

(a) Write the equations of motion for a point magnetic dipole moment $\vec{\mu}$ in a static uniform magnetic field $\vec{B}$. It is conventional to write $\vec{\mu} = g\frac{e}{2mc}\vec{s}$, where $\vec{s}$ is the "spin" angular momentum of the system.

(b) Show that $\vec{\mu}$ precesses around the direction of the magnetic field. What is the precession angular velocity?

(c) Consider of a solenoid with $N$ turns, radius $a$ and length $L$ that carries a current $I$. What is $\vec{B}(\vec{r})$ at very large distances from the center of the solenoid ($a, L \ll |\vec{r}|$)?

(d) Suppose a neutral particle with magnetic moment $\vec{\mu}$ travels slowly down the symmetry axis of the solenoid (from $z = -R$ to $z = +R$). Show that to leading order in $a/R$

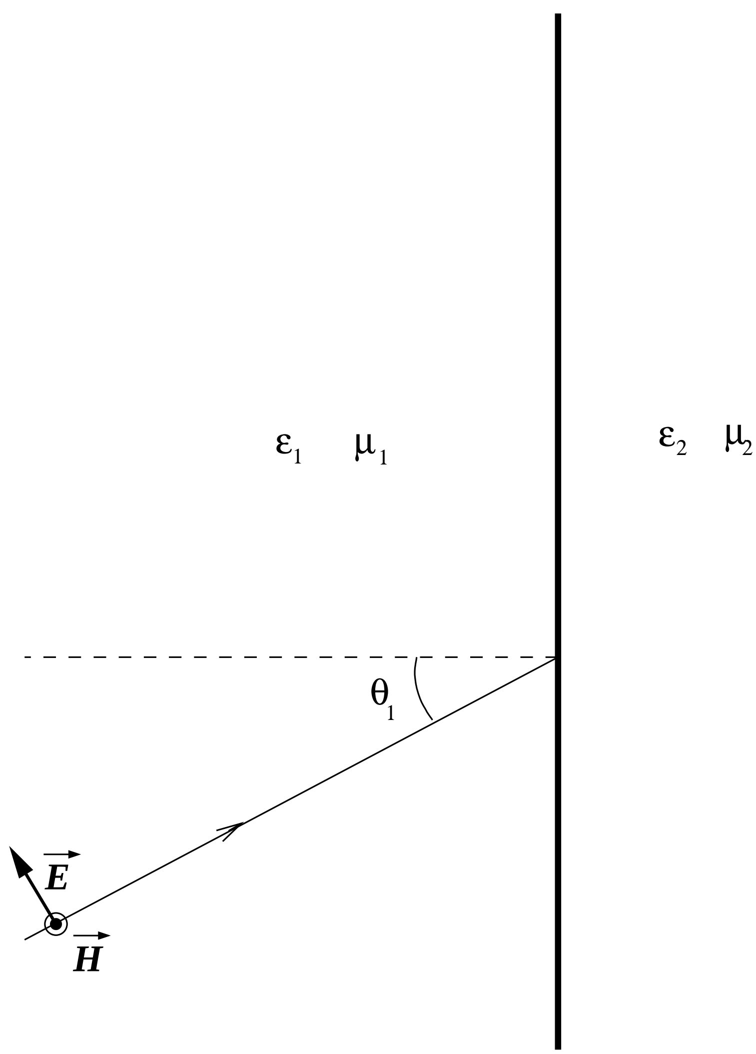
$$\int_{-R}^{R} (\vec{B} \cdot \hat{z}) \mathrm{d}z = \frac{4\pi NI}{c} \left( 1 - \frac{a^2}{2R^2} \right).$$

What is the net precession angle of $\vec{\mu}$?

(3) **Reflection and Refraction of TM Wave** – An electromagnetic plane wave is incident on the interface between two media in which both the permittivities $\varepsilon_1$ and $\varepsilon_2$ *and* the permeabilities $\mu_1$ and $\mu_2$ differ. The angle of incidence $\theta_1$ is the angle between the wave-normal and the normal to the interface (see figure). Take the wave to be polarized with the magnetic field normal to the plane of incidence (this is referred to as a transverse magnetic, or TM wave).

(a) Find the Brewster angle – the angle of incidence $\theta_B$ at which the reflection coefficient of the TM wave vanishes.

(b) Sketch $\sin^2\theta_B$ as a function of $\varepsilon_1/\varepsilon_2$. Argue that the TM wave has no Brewster angle for a certain range of values of the ratio $\varepsilon_1/\varepsilon_2$. Find the limits to this range in terms of $\mu_1$ and $\mu_2$.



(4) **Electromagnetic Wave Inside a Conductor**

(a) In the interior of a neutral conducting medium the charge density is $\rho = 0$ while the current density obeys Ohm's law, $\vec{J} = \sigma \vec{E}$. Note that, in general, $\sigma$ is a function of the electric field. By applying Maxwell's equations to a conductor derive the wave equation for $\vec{E}(\vec{r}, t)$ in such a medium. Write the general solution for a plane wave of frequency $\omega$ traveling in the $+z$ direction, and briefly describe the propagation of $\vec{E}$ for the case of a constant ($\omega$ independent), real $\sigma$ in the limiting cases $\omega \gg \sigma$ and $\omega \ll \sigma$.

(b) In a plasma of ionized particles, the electrons, with number density $n_e$, can be regarded as essentially free particles. Compute $\sigma(\omega)$ when a sinusoidally alternating electric field is applied to the plasma, and show that it is purely imaginary. Interpret the meaning of a purely imaginary conductivity.

(c) Show that, in a plasma (described according to your answer in (b)), electromagnetic waves will propagate only if $\omega > \omega_p$, the plasma frequency. Express $\omega_p$ in terms of $n_e$, the number density of electrons, and fundamental constants.