Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Wednesday, June 10 2009, 9 am - 1 pm

Electricity and Magnetism

Solve 3 out of 4 problems

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

1. (a) Show that electromagnetic waves propagate in vacuum; *i.e.*, that electric and magnetic fields satisfy wave equations.

(b) Suppose a plane polarized electromagnetic wave propagating in vacuum is normally incident on a half-space of electric permittivity $\epsilon = 3\epsilon_0$, and magnetic permeability μ_0. Find the fraction of the beam power which propagates into the dielectric (there is no absorption of the beam by the medium).

(c) If the total power carried by the beam is 2 W, find the force exerted by the incident beam on the dielectric.

2. Consider a sphere of radius R, whose interior has permanent, uniform electric polarization $\mathbf{P} = P\hat{\mathbf{z}}$ and electric permeability ϵ. The sphere is surrounded by vacuum.

(a) Find the **E** field in the interior of the sphere.

(b) Find the **E** field exterior to the sphere.

(c) Suppose that the sphere is placed so that its center is a distance ℓ from a conducting surface at $z = 0$ (the polarization charges in the sphere are immobile). What is the direction, and magnitude of the force on the sphere?

(d) Now suppose that the vacuum is replaced by a dielectric medium with electric permittivity $\epsilon' < \epsilon$ and magnetic permeability μ_0. Explain how the force of (c) changes.

3. Consider a small insulating ball of mass m which has an electric charge q placed on its surface, hanging from a spring of force constant k.

(a) The mass is displaced vertically from its equilibrium position by an amount x_0 and let go at $t = 0$, and it is observed to oscillate up and down at nearly constant frequency. Find the oscillation frequency (assume nonrelativistic motion and ignore electromagnetic effects).

(b) After a long time (many oscillation cycles) the motion is observed to slowly change. Explain how and why the motion changes.

(c) Find the oscillation amplitude as a function of time.

(you may obtain partial credit for (c) by using dimensional analysis to estimate the time scale on which the motion changes).

4. A long coaxial cable consists of a solid central conductor of radius a inside a conducting cylindrical shell of radius b. The inner cylinder carries charge per length $+\lambda$ and a battery drives a current I down the cylinder. The outer cylinder carries charge per length $-\lambda$, and a current I flowing in the opposite direction to the current in the inner cylinder.

(a) Calculate the **E** and **B** fields inside the cylinder.

(b) Calculate the Poynting vector **S** inside the coaxial cable (in the gap between the two cylindrical conductors), and the total linear momentum per unit length carried by the electromagnetic fields.

(c) Now, the battery is disconnected so that the currents stop flowing but the charges remain on the inner and outer cylinders. Calculate the net change in linear momentum per length imparted to the cylinders by the induced EMF, and compare to the result of (b).