Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Thursday, June 11, 2009, 9 am - 1 pm

Quantum Mechanics

Solve 3 out of 4 problems

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

(1) A particle of mass m moves in one dimension in a potential consisting of two δ functions,

$$V(x) \quad = \quad -V_0\left[\delta(x-a)+\delta(x+a)\right]$$

where V_0 is a real positive constant.

(A) Find the transcendental equation which the energy eigenvalues must satisfy (you don't need to solve it).

(B) Determine the conditions under which there exists a parity-odd bound state.

(2) A beam of electrons is prepared as a plane wave carrying momentum p, moving along the positive $\hat{x}$-axis. The beam is incident on a plate of width d oriented perpendicularly to the x-axis (so with its surface spanning the y-z planes) which provides a repulsive potential,

$$V(x) = \begin{cases} 0 & x < -d/2 \\ V_0 & -d/2 < x < d/2 \\ 0 & x > d/2 \end{cases}$$

For $E < V_0$ find the probability to tunnel into the region $x > d/2$.

(3) An electron bound by the electrostatic force to a proton in the ground state has binding energy,

$$E_{1s} \;=\; -\frac{e^4 m}{2\hbar^2}$$

and spherically symmetric wave function,

$$\psi_{1s}(r) \;=\; \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

where $a \equiv \hbar^2/(e^2 m)$ is the Bohr radius, and m and e are the mass and charge of the electron, respectively. For the purposes of this problem, neglect the electron and proton spins.

(A) Taking the finite size of the proton into account, we replace the proton by a sphere of uniform charge density with radius $R \ll a$. Compute the leading (in R/a) correction to the binding energy compared to the point-like proton case.

(B) Explain what effect the finite size of the proton has on the higher energy levels. For full credit, explain your answer quantitatively.

(4) A diatomic molecule in three dimensions is modeled as two point masses M connected by a spring of constant k. For the purposes of this problem, neglect any spin-spin interactions between the two atoms.

(A) Write down the Schrödinger equation which governs the energy eigenstates of the molecule. You do not need to solve it.

(B) Neglecting any degrees of freedom internal to the atoms themselves, explain which quantum numbers specify the state of the molecule and their physical meaning.

(C) Now consider the case in which the two atoms are *identical* spin 1/2 particles. What is the spin wave function for the ground state?