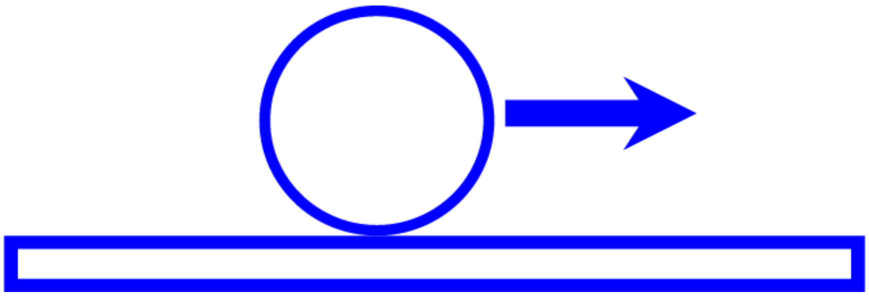
Department of Physics & Astronomy Qualifying Exam
21 September 2001, Morning

Classical Mechanics Problem 1



A cylinder of mass M, radius R, and moment of inertia $I = \frac{1}{2}MR^2$ is initially (at t=0) moving perpendicular to its axis while skidding (slipping) on a horizontal surface having coefficient of kinetic friction μ_K and coefficient of static friction μ_s. The initial velocity of the center of mass is v_i and its initial angular velocity is zero. The cylinder later begins to roll.

(i) What is its initial kinetic energy at $t = 0$?

(ii) What is the velocity of the center of mass and angular velocity at a time $t > 0$ but before it has begun to roll without skidding (i.e. when it is both rolling and skidding)?

(iii) What is the linear velocity of the cylinder's center of mass just as it begins to roll without skidding?

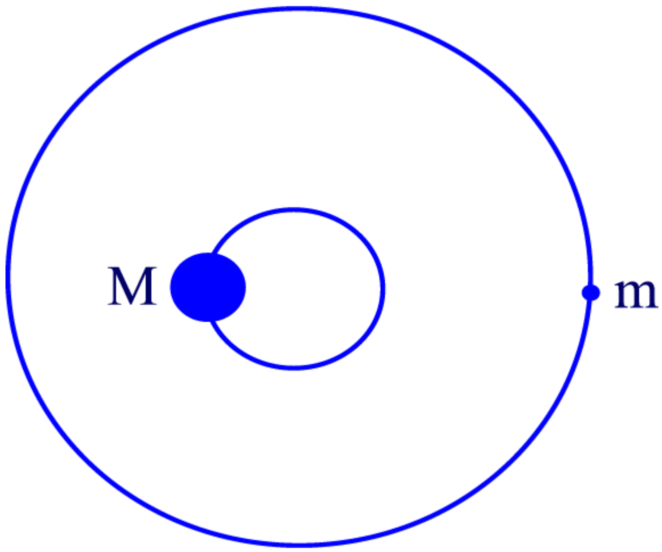
(iv) At what time, T, does it begin to roll without skidding?

(v) What is its total kinetic energy at this point?

(vi) At what rate was work being done by friction while the cylinder was skidding? Compare this answer with the answers to (i) and (v).

(vii) Calculate the cylinder's angular momentum as a function of time.

Classical Mechanics Problem 2



Consider a star of mass M and a satellite of mass m in (different) circular orbits. Let the satellite be a point mass and the stellar object characterized by a moment of inertia, I. Assume that the star spins at frequency ω in the same sense as their orbital motion, as the two masses revolve about their common center of mass at frequency Ω.

a) Show that the orbital separation, a, between the star and its satellite is $a = \left[\frac{G(M+m)}{\Omega^2}\right]^{1/3}$

b) What is the energy of the system?

c) What is the angular momentum of the system?

d) Show that for a given total angular momentum L, the energy is minimized at synchronism, i.e. when $\omega = \Omega$, for sufficiently small Ω.

e) Find the condition on Ω for the equilibrium in part (d) to be stable. Give your answer in terms of I and the orbital moment of inertia $\frac{Mm}{M+m}a^2$