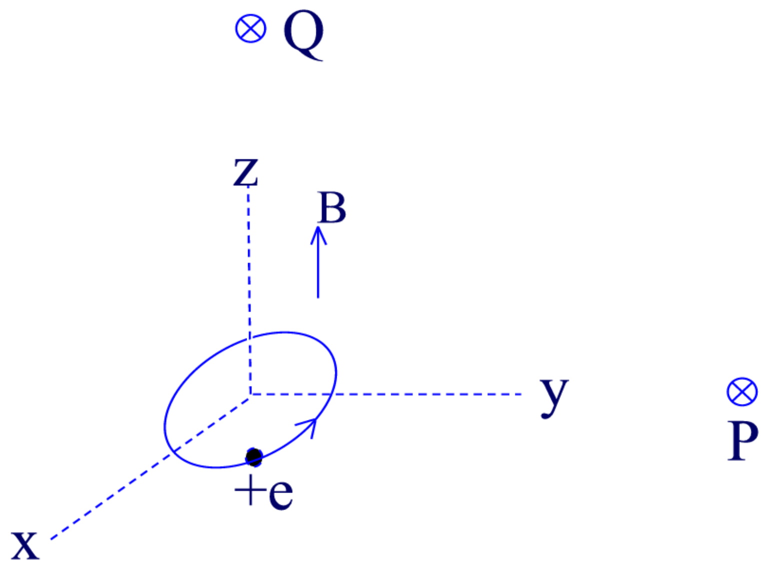
Department of Physics & Astronomy Qualifying Exam
20 September 2001, morning

Electricity and Magnetism Question 1

(State whether you are working in the SI or Gaussian System)

A positron (charge + e, mass m) is moving in a circular orbit in the xy plane in a uniform magnetic field $\vec{B} = B_O \hat{z}$. The positron speed $v \ll c$.



i. Find the frequency ω of the positron orbit and show that it is independent of r, the radius of the orbit. Find ω for $B_0 = 1$ T (1 T ≡ 10^4 G).

ii. The above discussion neglects radiation. Describe the polarization of the radiation as seen by observers at points P and Q (along $\hat{y}$ and $\hat{z}$ axes, respectively). Take P and Q to be in the far zone, and consider only electric dipole radiation.

iii. Find the total power radiated assuming that the orbit stays very close to circular at all times. In other words, assume the positron spirals into the origin extremely slowly.

You do not have to be accurate to factors of order unity, but you must get all dimensional factors correctly. Dimensional analysis may help.

iv. Let the kinetic energy of the positron be ε(t). By equating the rate of decrease of ε(t) to the power from (iii) and continuing to assume a quasicircular orbit, show that

$$\frac{d\varepsilon}{dt} \approx -\frac{\varepsilon}{\tau}$$

v. Find an expression for τ in terms of m, c, e and ω and give its value for $B_o = 1$ T. Is the assumption of an quasicircular orbit justified?

Electricity and Magnetism Question 2

(State whether you are working in the SI or Gaussian System)

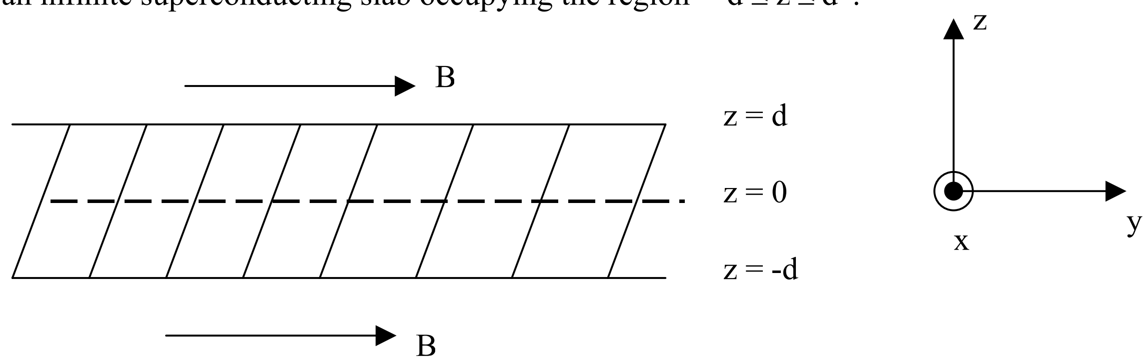
In a superconductor, a current distribution $j(\vec{r},t)$ is related to $\vec{E}$ and $\vec{B}$ fields via

$$\vec{E} = \lambda \frac{\partial}{\partial t} \vec{j} \qquad \text{(SI and Gaussian)}$$

$$\vec{B} = \begin{cases} -\lambda c\left(\vec{\nabla} \times \vec{j}\right) & \text{(Gaussian)} \\ -\lambda\left(\vec{\nabla} \times \vec{j}\right) & \text{(SI)} \end{cases}$$

Here λ is a material constant, c is the speed of light, and $\vec{E}$ and $\vec{B}$ are macroscopic fields (i.e. spatial averages of the microscopic fields $\vec{e}$ and $\vec{b}$ over a length scale of 20 Å, say).

Consider an infinite superconducting slab occupying the region $-d \le z \le d$.



Outside the superconductor (both z > d and z < -d)

$B_x = B_z = 0$; $B_y = B_o$ = constant
$\vec{E} = 0$ everywhere

i. Find a differential equation for $\vec{B}$ inside the superconductor.

ii. Solve the equation in in part (i) for the geometry given.

iii. Obtain $\vec{j}$ everywhere inside the superconductor using your answers for parts (i) and (ii)

iv. Show that if d is large, $\vec{B}$ is very small at z = 0 and effectively penetrates only up to a characteristic distance Λ from the surfaces z = ± d. Find Λ.

v. Sketch your answers for $\vec{B}$ and $\vec{j}$ using labelled and informatively marked graphs, assuming d > > Λ.