Department of Physics & Astronomy Qualifying Exam
21 September 2001, Afternoon

Quantum Mechanics Question 1

Assume a particle moving in one-dimension is bound in a potential well of the form

$$V(x) = -C|x|$$

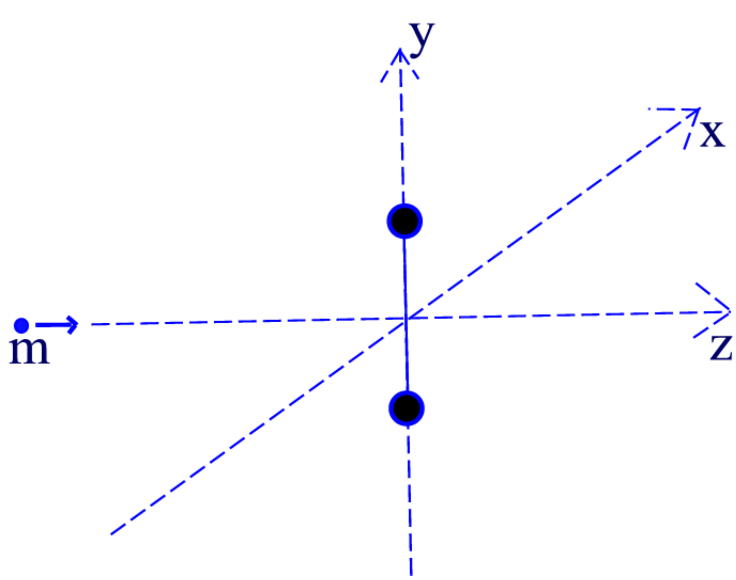
where C is a positive constant. Using a trial wavefunction of the form

$$\psi(x) = Ae^{-\alpha x^2},$$

calculate the best approximation to the ground state energy.

(Note $\int_0^{+\infty} dx e^{-\alpha x^2} = \frac{1}{2}\left(\frac{\pi}{\alpha}\right)^{1/2}$.)

Quantum Mechanics Question 2



Consider the scattering of a spinless particle of mass m traveling parallel the $\hat{\mathbf{z}}$ axis with a momentum k from a fixed diatomic molecule with its axis parallel to $\hat{\mathbf{y}}$. Assume the interaction potential between the neutron and the diatomic molecule is given by

$$V(x,y,z,) = V_0[\delta(y-y_0)+\delta(y+y_0)]\delta(x)\delta(z).$$

Part (a).

Using the Born approximation, calculate the scattering amplitude, $f(\theta,\phi)$,

Part (b).

Calculate the resulting differential scattering cross section, $\frac{d\sigma(\theta,\phi)}{d\Omega}$.

Quantum Mechanics Question 3

Part (a).

Assume that the proton (in the H atom) is not a point charge, but is rather spread uniformly over a sphere of radius R. Thus the potential energy is given by

$$V(\mathbf{r}) = -\frac{3e^2}{2R^3}\left(R^2 - \frac{1}{3}r^2\right) \qquad (r < R)$$

$$V(\mathbf{r}) = -\frac{e^2}{r} \qquad (r > R)$$

(assume where possible that R << a, the Bohr radius). Calculate the shift in energy of the 1s level. $\left(\text{note } \psi_{1s} \propto e^{-r/a}\right)$

Part (b).

Rather than the potential in part (a), now assume that the electron and proton interact with the following spin-dependent potential energy

$$V(\mathbf{r}) = -\frac{e^2}{r} + V_0 \hat{\mathbf{s}}_e \cdot \hat{\mathbf{s}}_p \delta^{(3)}(\mathbf{r})$$

where $\hat{\mathbf{s}}_e$ and $\hat{\mathbf{s}}_p$ are the spin operators of the electron and proton respectively. Calculate the energy difference between the singlet and triplet energy levels of the 1s state.