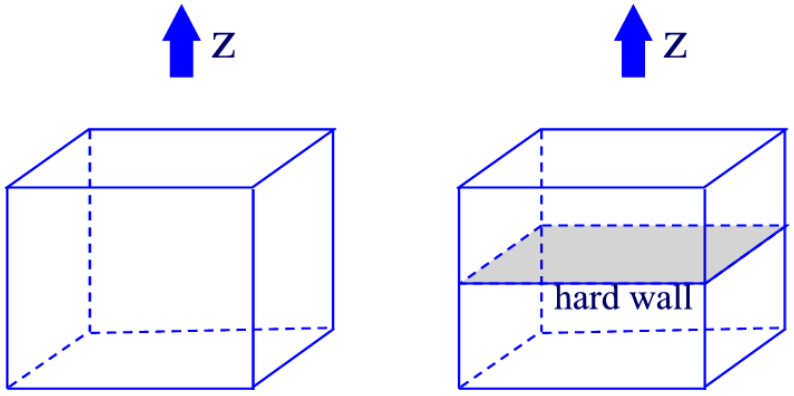
Department of Physics & Astronomy Qualifying Exam
20 September 2001, Afternoon

Statistical Mechanics Question 1

A solid contains N atoms, each with spin 1 and magnetic moment μ, in an uniform magnetic field B.

Determine an expression for the mean magnetic moment M at temperature T, assuming that the dipoles do not interact with each other. Also find the susceptibility per atom $\chi \equiv \frac{1}{N}\frac{\partial M}{\partial B}$ in the limits $\mu B << k_B T$ and $\mu B >> k_B T$.

Statistical Mechanics Question 2



A gas of nonrelativistic, non-interacting electrons (mass m) is at zero temperature and contained in a large cube of side L. The density is ρ electrons per unit volume.

(a) Assuming periodic boundary conditions, determine the maximum kinetic energy E_F.

(b) Count the number of occupied states that have $K_z=0$. Sum their energies and express the result in terms of E_F, L and fundamental constants.

(c) Consider a horizontal *impenetrable* wall to be inserted such as to divide the cube in two equal halves, thus creating additional surface area $2L^2$. Are the states counted in (b) still allowed? (Explain.) Assuming that any electrons displaced from their previous states must now occupy states approximately at the Fermi energy, calculate the work required per unit area to insert the wall.

[SEE OTHER SIDE]

Statistical Mechanics Question 3

A classical gas (N particles of mass m, temperature T) is in a cylindrical container of large radius R and height L that is spinning about its axis with angular velocity ω. Thus each particle experiences a centrifugal force $m\omega^2 r$ where r is the distance of the particle from the axis of rotation.

Calculate the number density as a function of r. Do this calculation in the limit of negligible interactions, i.e. assuming an ideal gas.