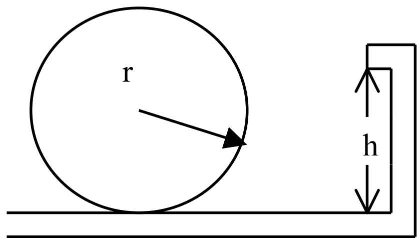
Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 1

A billiard ball (radius r) is rolling towards the right without slipping, at a linear velocity v_i . The moment of inertia of a solid sphere is $2mr^2/5$.



(a) Relate the linear momentum to the angular momentum.

(b) The ball hits the edge of the billiard table (height h) and rebounds with velocity v_f (moving to the left). It is found that the ball is still rolling without slipping. Find h in terms of r.

Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 2

A large mass, M_0, holds a small mass, m, in a circular gravitational orbit at radius r_0. Here, $m \ll M_0$.

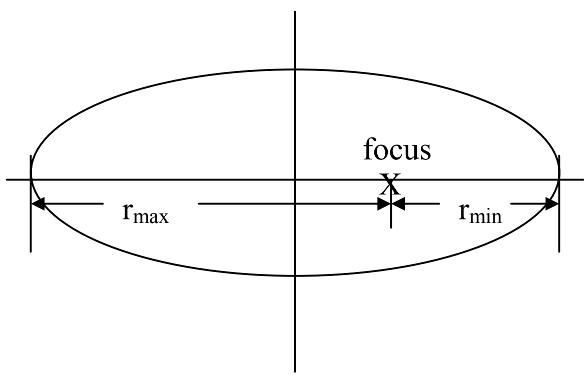
(a) Express the kinetic energy, K_0, and the total energy, E_0, in terms of the potential energy U_0.

(b) Suppose the mass M_0 is suddenly decreased to $M = fM_0$, where $f < 1$. Express the new angular momentum, L, and energy, E, in terms of the parameter f and the parameters of the original orbit.

(c) What is the largest value of f at which the system becomes unbound?

(d) For general f, give the semi-major axis and eccentricity of the new orbit in terms of f and r_0.

One of the definitions of an ellipse's eccentricity, ε, is given in the figure.

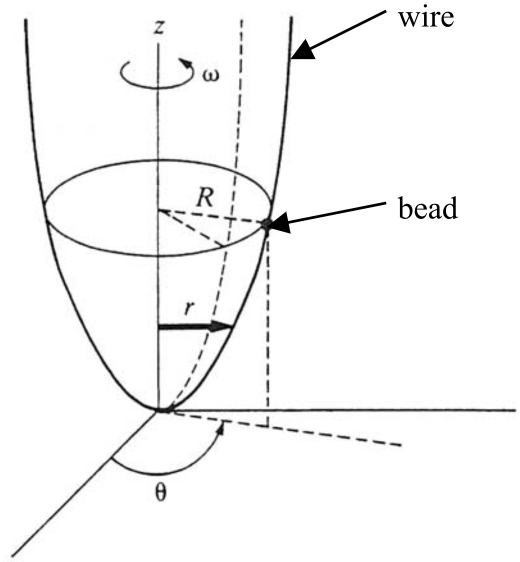


$$\frac{r_{min}}{r_{max}} = \frac{1-\varepsilon}{1+\varepsilon}$$

Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 3

A bead slides along a smooth wire bent in the shape of a parabola $z = cr^2$, under the influence of gravity.



(a) Write the Lagrangian for this system.

(b) The bead rotates in a circle of radius R when the wire is rotating about its vertical symmetry axis with angular velocity ω. Find the value of the parameter c.

Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 4

Consider a collection of N identical independent distinguishable harmonic oscillators of frequency ω.

Determine the energy per oscillator (E/N) as a function of the temperature T.

Obtain an expression for the specific heat of this system and write the asymptotic form in the $T \to 0$ and $T \to \infty$ limits.

Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 5

Consider a superdense gas of N electrons in a volume V. Assume that the dependence of energy upon momentum is extremely relativistic.

(a) Find the chemical potential of this gas in the limit of zero temperature ($T = 0$).

(b) Find the pressure in the $T = 0$ limit.

(c) Obtain a condition on the density of the gas, such that the assumption that the gas can be regarded as extremely relativistic is justified.

Department of Physics & Astronomy
Classical and Statistical Mechanics Qualifying Exam

Problem 6

Consider N non-interacting spins, each with s = 1/2, and magnetic moment μ in an external magnetic field H, and at temperature T.

(a) Show that the magnetization, M, is given by

$$M = \mu \tanh \frac{\mu H}{2k_B T}$$

(b) Suppose we have such a body in equilibrium at an initial temperature, T_{in}, and magnetic field H_{in}. Further suppose that it is isolated from its environment and then the magnetic field is reduced from H_{in} to a (lower) value H_{final}; it is assumed that the body remains in internal thermal equilibrium during this process. Obtain an expression for the final temperature.