Department of Physics & Astronomy
Qualifying Exam
Electricity and Magnetism

Problem 1

(a) Consider a neutral plasma with n electrons per unit volume. The electrons are much more mobile than the ions, and may be taken to be the only contributors to the current.

Find the dispersion relation (formula relating frequency, ω, and wave vector, k) for EM waves in this plasma. Express your result in terms of the plasma frequency ω_p where

$$\omega_p^{\ 2} = \frac{4\pi ne^2}{m} \text{ (Gaussian) } ; \quad \frac{ne^2}{m\varepsilon_0} \text{ (SI)}$$

(b) Suppose that the plasma is in a strong static uniform magnetic field $\vec{B}_0$. Find the dispersion relation for left and right circularly polarized EM waves traveling parallel to $\vec{B}_0$. Express your result in terms of ω_p and ω_B, where

$$\omega_B = \frac{eB_0}{mc} \text{ (Gaussian); } \quad \frac{eB_0}{m} \text{ (SI).}$$

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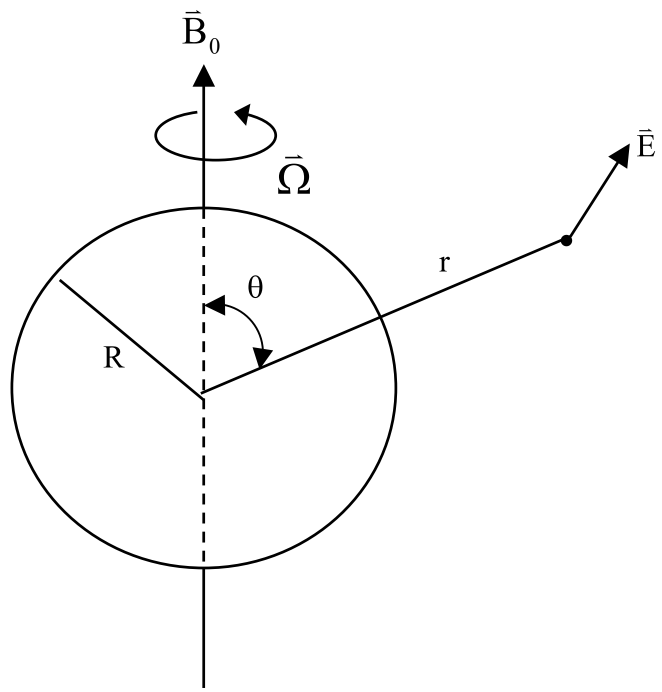
Problem 2

An uncharged, non magnetic, conducting sphere rotates with constant angular velocity, $\vec{\Omega}$, about a diameter in a uniform magnetic field, $\vec{B}_0$, with $\vec{B}_0 \parallel \vec{\Omega}$. Assume the radius of the sphere is R.



(a) Using the fact that there is no net force on a charge in the interior of the sphere, calculate the electric field, $\vec{E}_{in}$, inside the sphere. Find the charge distribution inside the sphere. What is the potential on the surface of the sphere?

(b) Find the electric field, $\vec{E}_{out}$, outside the sphere at a distance r and angle θ (see figure). (Hint: Determine the potential, ϕ_{out}, and derive the electric field).

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Problem 3

You are given a medium of infinite extent in which there is a *uniform* electric polarization $\vec{P}$ (you may think of a ferroelectric material which can have a spontaneous polarization even in the absence of any applied fields).

(a) Assume that an infinitely long cylindrical cavity is present within the medium. The axis of the cavity is parallel to $\vec{P}$. Calculate the electric field on the axis of this cylinder. (Here assume that the formation of the cavity does not alter the polarization or its direction in the surrounding medium.) Will the electric field be uniform within this cavity?

(b) Repeat part (a) assuming the axis of the cavity is perpendicular to $\vec{P}$.

(c) Now assume that a spherical cavity is present in the medium. What will the electric field be at the center of this sphere? Will the electric field be uniform within the cavity?

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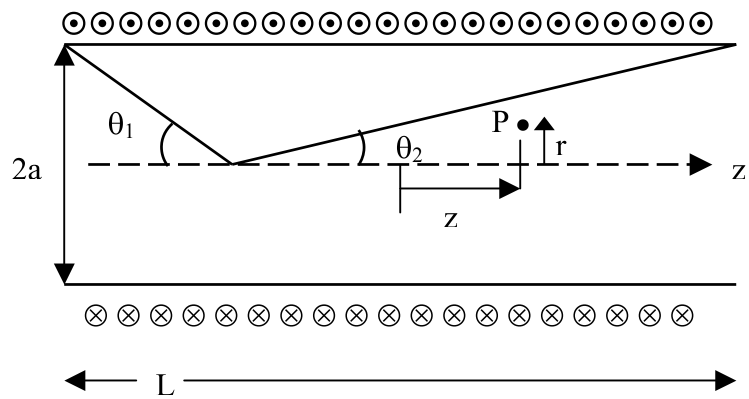
Problem 4



Consider a long thin solenoid of length L, circular cross section of radius a. It is tightly wound with N turns per unit length, carrying a current I.

(a) Show that on the axis, the magnetic intensity is given by

$$\vec{B} = K(\cos\theta_1 + \cos\theta_2)\hat{z}$$

where θ_1 and θ_2 are defined in the figure, and K is a constant to be found.

(b) Show that at a point P <u>near the symmetry axis</u> and <u>near the center</u> of the solenoid, $\vec{B}$ is mainly parallel to the axis as in part (a), but also has a small radial component

$$B_r = K'\left(\frac{a^2 zr}{L^2}\right)$$

correct to order a^2/L^2 for $z \ll L$, and $r \ll a$. Find the constant K'.