Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Thursday, September 20, 2012, 9 am - 1 pm

Electricity and Magnetism

Solve 3 out of 4 problems

If you solve all 4 problems, only the first 3 will be graded.

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

1. Two beams of light are drawn from the same point source. One beam is incident at an oblique angle on a plane mirror as shown in the diagram. The beams are plane-polarized in the plane of the mirror. The beam intensities are arranged so that the individual electric fields have the same amplitude at point P.

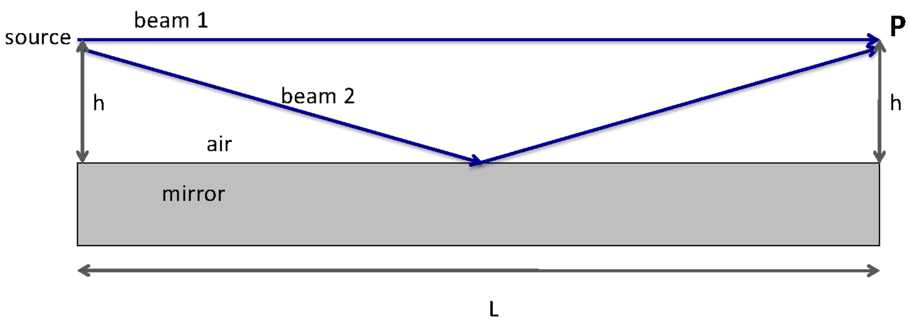
   (a) What is the intensity I at point P as a function of h and λ? (Express $I(h, \lambda)$ with respect to a maximum intensity I_{max}.)

   (b) Explain how this device [1] can be used to measure λ.

   (c) Show that I vanishes as h^4 for $h \ll \sqrt{\lambda L}$.

   (d) If $L = 1$ m, what is the separation Δh between the first maxima for red ($\lambda = 700$ nm) and for blue ($\lambda = 400$ nm) light?

   (e) A small, perfectly absorbing screen is placed at point P, with area A, and is entirely illuminated by the beams, which strike it at near-normal incidence.
   What is the force exerted on the screen when I is a maximum, and how does it compare to the force obtained if beams 1 and 2 lose coherence?



[1] This is known as Lloyd's mirror and was in fact used to measure λ for light.

2. A mass M swings on a pendulum of length L; the natural frequency of the motion is $\omega_0 = \sqrt{g/L}$.

   A charge Q is placed on the mass. The time variation of the acceleration of the mass causes a radiative reaction force:

$$\vec{F}_{\text{rad}} = M\tau\dot{\vec{a}}$$

   where $\tau = Q^2/(6\pi\epsilon_0 c^3 M)$ has the dimensions of time.

   The damping caused by F_{rad} is slight: $\omega_0\tau \ll 1$.

   How many periods, N, must go by before the energy of the pendulum decreases by a factor of $1/e$?

   (Note: You may want to make the approximation $\omega^3\tau \approx \omega_0^2\omega\tau$.)

3. A conducting sphere of radius a has charge Q at $t = 0$. A homogeneous and isotropic dielectric material encases the sphere; it is also shaped like a sphere and has radius $2a$ (see diagram) and dielectric constant ϵ.

   (a) Find $\vec{E}$ everywhere and plot $|\vec{E}|$ as a function of r.

   (b) What work is done in increasing the charge on the conductor from Q to $Q + dQ$?
   Bring your answer into a simple form.

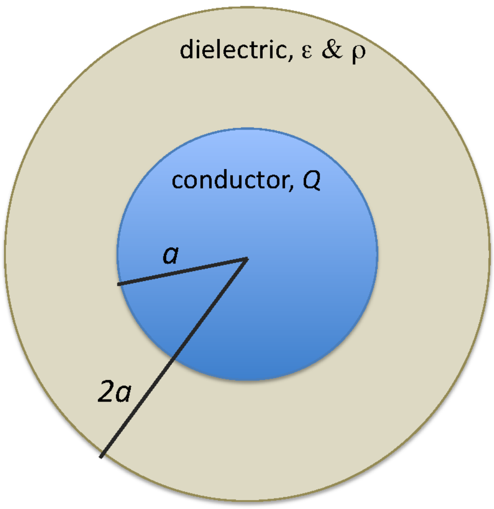
   (c) What is the capacitance of the conducting sphere + dielectric?

   (d) The dielectric is "leaky" and charge escapes from the conductor at a small rate. If the constant resistivity of the dielectric is ρ, find the resistance R.

   (e) What is the time, τ, needed for the charge on the conductor to decrease by a factor of $1/e$?
   What is the dependence of τ on the radius a?

   (f) Find $\vec{E}$ everywhere for $t \to \infty$.

   (g) Calculate the initial and final energy and explain any difference.



4. A sinusoidal current is established on a long, thin wire stretched along the z-axis. Let the perpendicular distance from the z-axis be s.

   The current forms a *standing wave*:

$$I(z,t) = I\,\cos(kz)\sin(\omega t)$$

   where $k = 2\pi/\lambda$. We will consider the quasi-static case, in which $|z|$ and s are much less than c/ω.

   (a) What is the magnetic field, $\vec{B}(s,t)$ at $z = 0$?
   Do not forget to specify the direction of the magnetic field.

   (b) A rectangular open loop is placed in the zx plane as shown in the figure. It has length $\ell \ll \lambda$ in the z-direction and height h in the x-direction. It is centered on $kz = 0$. What is the time-dependent potential difference $V(t)$ across the gap?
   Make appropriate approximations for the z-dependence of $\vec{B}$.

   (c) Move the rectangular loop in the $+\hat{z}$ direction to $kz = \pi/2$. What happens to $V(t)$?

   (d) Suppose parallel plates are attached to the open points of the loop. Each plate has area A and they are separated by distance d. What is the time-dependent charge, $Q(t)$, on one of the plates, in terms of ω, A, d and ℓ?

   (e) Remove the plates and close the loop. What is the mutual inductance M of the loop with the wire?