Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Monday, June 8, 2015, 9 am - 1 pm

Electricity and Magnetism

Solve 3 out of 4 problems

If you solve all 4 problems, only the first 3 will be graded.

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

Note that you may use the Gaussian or SI systems of units for any problem, but please use the same system throughout the entirety of each problem.

1. Consider an uncharged conducting sphere of radius a, with its center at the origin $\mathbf{r} = \mathbf{0}$. The sphere is in an electric field which tends to the limit $\mathbf{E} = E_0\hat{\mathbf{z}}$ for $r \gg a$.

(a) Find the electric field throughout space. What is its leading multipole component?

(b) Find the surface charge distribution on the sphere.

(c) What is the force (magnitude and direction) due to the electric field on the *upper half* of the sphere (the $z > 0$ half)?

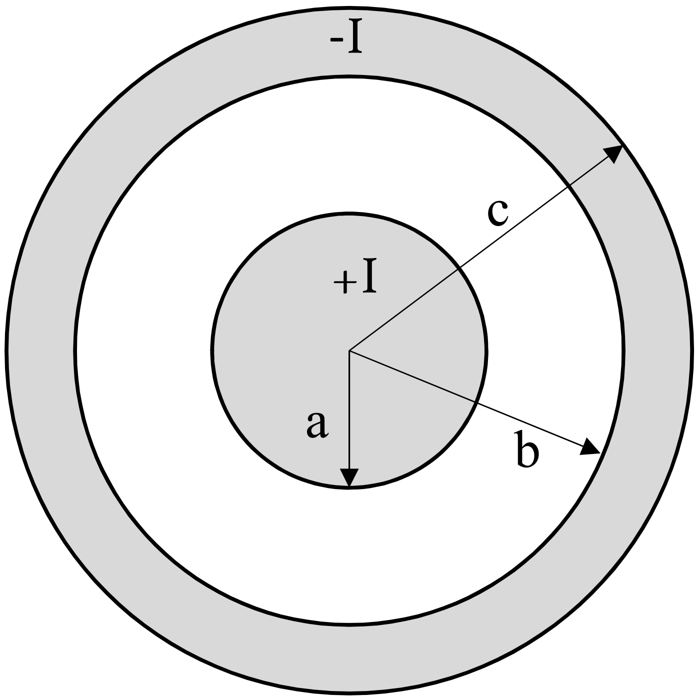
2. Consider a long, conducting cylindrical pipe of inner radius b and outer radius c ($b < c$), which carries current I in the $-z$ direction, with constant current density across its cross-section. In the middle of the pipe is a conducting cylinder of radius a which also carries current I but in the $+z$ direction, with constant current density across its cross-section. The magnetic permeability in the conductors equals that of the vacuum.

(a) Find the magnetic field in all regions.

(b) Suppose the pipe and inner cylinder each carry equal and opposite charge per unit length ($-\lambda$ on the pipe and $+\lambda$ on the inner cylinder).
Find the electric potential in the region between the conductors.

(c) Find the electromagnetic field energy density in the region between the central conductor and the pipe ($a < r < b$) as a function of I and λ.

(d) What are the capacitance and the inductance per unit length of the pipe/cylinder combination?



3. Linearly polarized light of frequency ω is incident in vacuum onto a dielectric that fills the half-space $z > 0$. At this frequency the dielectric constant is the real number ϵ. The direction of propagation $\hat{\mathbf{s}}$ makes an angle θ with the normal $\hat{\mathbf{n}}$ to the surface of the dielectric. The direction of the electric field of the incident beam is parallel to (in the plane of) the surface (in the x-y plane), and the amplitude of the electric field of the incident beam is E_i.

(a) Use boundary conditions to determine the directions of propagation and the electric field amplitudes of the reflected and transmitted waves.

(b) Compute the power in the reflected and transmitted beams.

(c) Is energy conserved by the conversion of the incident beam to reflected and transmitted beams? Prove your statement using your results from (b).

Recall that as one crosses the interface between two linear dielectric media, $\hat{\mathbf{n}} \cdot \mathbf{D}$ and $\hat{\mathbf{n}} \times \mathbf{E}$ are continuous, where $\mathbf{n}$ is the surface normal of the interface. You may assume that the magnetic permeability of the dielectric is the same as that of the vacuum, so that $\mathbf{B}$ and $\mathbf{H}$ are continuous everywhere. Also note that $\mathbf{D} = \epsilon_0 \epsilon \mathbf{E}$ in SI units (in Gaussian units $\mathbf{D} = \epsilon \mathbf{E}$).

4. An insulating ball of mass m and negligible radius carries charge $+Q$ and hangs by an insulating string of length ℓ, pulled down by gravitational acceleration g in the z-direction.

(a) Find the frequency of oscillation of the ball if it is released with a small amplitude ($d \ll \ell$) in the x-direction.

(b) Find the condition for the oscillatory motion to be non-relativistic. You should assume this condition to hold for the remainder of the problem.

(c) Suppose the ball swings with amplitude $d \ll \ell$ in the x direction, and with zero amplitude in the y direction. Find the (radiation) fields far from the pendulum. You may neglect the motion in the z direction.

(d) Based on radiated energy, find the number of oscillations that the ball will make before its amplitude decays significantly (*i.e.*, by a factor of $1/e$). Find a condition expressing the range of ℓ under which many oscillations occur.