Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Thursday, June 13, 2013, 9 am - 1 pm

Electricity and Magnetism

Solve 3 out of 4 problems

If you solve all 4 problems, only the first 3 will be graded.

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

1. A thin membrane is uniformly embedded with dipolar molecules such that all dipoles are aligned perpendicular to it, and the dipole moment per unit area is η. Consider a piece of this membrane in the shape of a disk of radius a. Let the normal to the disk and the direction of the dipoles both be **z**.

(a) Find the electric potential everywhere on the axis of the disk on both sides of the membrane.

(b) Find the electric field everywhere on the axis of the disk on both sides of the membrane.

(c) Find the net work done in transporting a unit test charge, first from $z = 0+$ to $z = \infty$, then in a very large semicircle from $z = \infty$ to $z = -\infty$, and then from $z = -\infty$ to $z = 0-$.

(d) Find the potential difference between the surfaces on opposite sides of the membrane.

2. A very large sheet of uniform thickness d is made of a metal of resitivity ρ.

(a) An electrode of essentially zero width is attached to the middle of this sheet, and a current I is injected into the sheet. Treating the sheet as infinite, and assuming that all the current leaves the sheet at the edges (so there is no build up of charge anywhere), find the current density everywhere in the sheet.

(b) Now suppose that there are two such electrodes, A and B, attached to two points on the sheet with x-y coordinates $(\pm 2a, 0)$, where $a >> d$. A current I enters at electrode A, and leaves at B. What is the current density everywhere in the sheet?

(c) Two more electrodes C and D are attached at $(\pm a, 0)$. What is the potential drop from C to D, assuming that any current leaving or entering at C and D is negligibly small?

3 (a). Two electrons in an electron beam are side-by-side (in the same plane transverse to the beam) a distance a apart, travelling at a uniform relativistic velocity v. What is the force on each electron (magnitude and direction) as measured in the lab frame? It is not sufficient to merely write down your answer. You must explain your calculation.

(b) What is the force on the electrons, as measured in the lab frame, if one is directly behind the other by a distance a as measured in the lab? Again, you must explain your calculation.

4. An electron is undergoing nonrelativistic cyclotron motion in a circle of radius *a* at a frequency ω_c in the *xy* plane.

(a) Starting from the expression for the instantaneous power radiated by an accelerating charge, find the total energy radiated per orbit.

(b) Find the time-averaged angular distribution of the radiated power. Note that this distribution must be cylindrically symmetric.

(c) What is the polarization of the radiation along the *z* axis? What is the polarization in the *xy* plane at an angle θ to the *x* axis? Give reasons for your answer.

(d) Assuming that the energy loss in part (a) is much smaller than the average energy over the orbit, find and solve a first order differential equation for the way in which the radius of the orbit changes.