Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Monday, June 9, 2014, 9 am - 1 pm

Electricity and Magnetism

Solve 3 out of 4 problems
If you solve all 4 problems, only the first 3 will be graded.

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

Problem 1

(a) Two lines with fixed charge densities $+\lambda$ and $-\lambda$ lie parallel to the z-axis and go through points $(a, 0, 0)$ and $(-a, 0, 0)$, respectively. Find the electric potential everywhere.

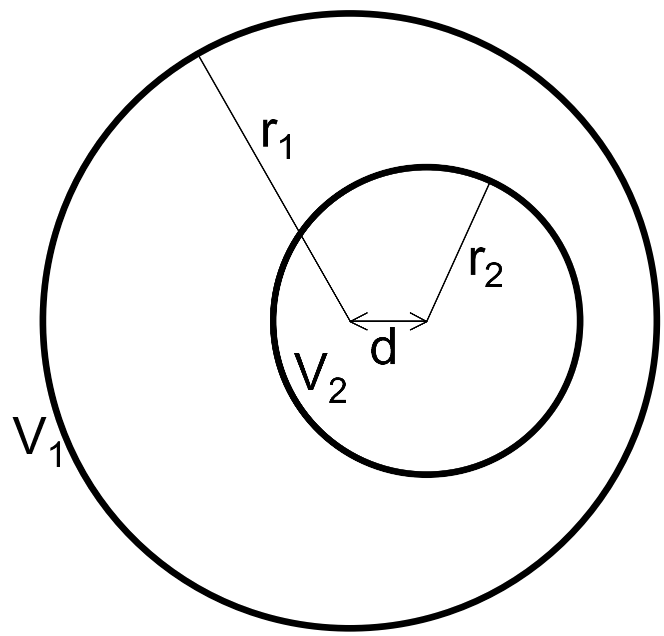
(b) Show that the equipotential surfaces from part (a) are circles in the x-y plane. Give an expression for the center and radius of an equipotential surface in terms of its voltage, and sketch a set of equipotentials.

Hint: You may find it convenient to define some dimensionless quantity, and to express other values in terms of that quantity.

Now consider the figure below, which shows two parallel, nested conducting cylinders with radii r_1 and r_2 and voltages V_1 and V_2, with axes *offset* from each other by distance d.

(c) Use your solutions from (a) and (b) to construct the potential in the space between the two cylinders. A set of equations relating V_1, V_2, r_1, r_2, and d to parameters in (a) and (b) is sufficient — you do not need to solve these equations.

(d) What is the capacitance per unit length of the system in part (c)? Justify your answer. You may write your answer in terms of any variables you defined in parts (a)-(c).



Problem 2

A cylindrically symmetric, charged top is spinning about its axis with angular velocity ω. The top's charge density is everywhere proportional to its mass density.

(a) What is the magnetic dipole moment $\vec{m}$ of the spinning top about the center of mass, in terms of its total charge, mass, and angular momentum?

The spinning top is placed in a uniform magnetic field $B_0\hat{z}$, such that the top precesses with angular frequency $\Omega \ll \omega$ at constant angle θ to the magnetic field. *There is no gravity.*

(b) Derive the precession frequency Ω of the top in the magnetic field.

(c) A loop of wire with radius a is placed a distance d from the top, where d is large compared to the size of the top and $a \ll d \ll c/\Omega$. To lowest order in a/d, what are the amplitude and frequency of the induced voltage around the loop when the loop is placed

(i) on the z-axis, loop axis along the z-axis

(ii) on the z-axis, loop axis along the x-axis

(iii) on the x-axis, loop axis along the z-axis

(iv) on the x-axis, loop axis along the x-axis

Potentially useful formulae:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[3\left(\vec{m}\cdot\hat{r}\right)\hat{r} - \vec{m}\right]$$

Problem 3

Consider a perfectly conducting rectangular waveguide along the z-axis with sides of length a and $2a$ ($\vec{E}$=0 and $\vec{B}$=0 in the conductor).

(a) Write down the boundary conditions for $\vec{E}$ and $\vec{B}$ inside the waveguide.

(b) Find all solutions to Maxwell's equations inside the waveguide with time and z-dependence of the form $e^{i(kz-\omega t)}$.
Hint: You will have two classes of solutions, one where $E_z = 0$ and one where $B_z = 0$.

(c) What are the phase and group velocities for each mode in your solution to (b)?

(d) What is the range of ω for which there is precisely one mode that propagates down the waveguide?

Problem 4

A parallel plate capacitor with plates of area A separated by distance d initially holds a charge Q_0. At $t = 0$, the two plates are connected by a wire with resistance R.

(a) Ignoring radiative losses, what is the charge on the capacitor as a function of time, for time $t > 0$?

(b) The discharging capacitor produces electric dipole radiation. Find the $\vec{E}$ and $\vec{B}$ fields in the radiation zone as a function of retarded time t_r. Assume all dimensions of the capacitor and resistor are much smaller than $c\tau$, where τ is the discharge time constant.

(c) What is the power lost through dipole radiation, as a function of time?

(d) What is the ratio of the radiated power to the power lost in the resistor? In part (a) we ignored radiative losses — is that a good approximation in this problem?