Northwestern University Physics and Astronomy Ph.D. Qualifying Examination

Thursday, June 10, 2010, 9 am - 1 pm

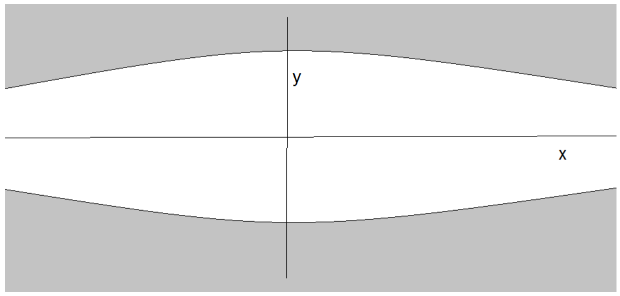
Quantum Mechanics

Solve 3 out of 4 problems

If you solve all 4 problems, only the first 3 will be graded.

Solve each problem in a separate exam solution book and write your ID number – not your name – on each book. If your solution uses more than one book, label each book with "1 out of 2", "2 out of 2" and so on.

QM Problem 1



A particle moves in a two-dimensional infinitely deep well described by the potential

$U(x,y) = 0 \qquad |y| \le \frac{a^2}{\sqrt{x^2+b^2}}$ where a << b; $\qquad U=\infty$ elsewhere.

a) A classical particle in this well with finite positive energy can escape to infinity by moving along the x-axis. Is the same true for a quantum particle? Is the energy spectrum of the quantum particle continuous? Explain qualitatively.

b) Because a<<b, the motion of the particle can be represented as a superposition of a fast motion along the y-axis and a slow motion along the x-axis. Use the adiabatic approximation as follows. First solve the Schrödinger equation for a fixed value of x (i.e. neglecting the x-derivative in the Hamiltonian). Find the corresponding eigenfunctions $\psi_n(x,y)$ and $E_n(x)$. Then try to look for solutions to the full two-dimensional Schrödinger equation in the form $\Psi(x,y) = \phi(x)\, \psi_n(x,y)$. Assuming slow variation of $\psi_n(x,y)$, derive an equation for $\phi(x)$ and solve it. Do you get a continuous or discrete spectrum this way?

QM Problem 2

A 1-dimensional harmonic oscillator with mass m, charge e and classical frequency ω is in its ground state $\psi_o = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha x^2}{2}}$ where $\alpha \equiv \frac{m\omega}{\hbar}$.

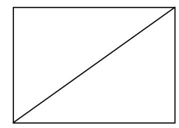
a) Find the wave function for the first excited state.

b) A weak electric field E is turned on at time t=0 and off at t=T. Estimate the probability that the oscillator is excited to its first excited state.

QM Problem 3

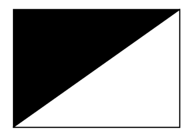
A particle of mass μ is confined in a square two-dimensional box: $0 \leq x \leq a$ and $0 \leq y \leq a$.

a) Write the eigenfunctions of the Schrödinger equation and find the corresponding energy levels. What degeneracies occur because of the square symmetry?



(b) A small potential $V_o\delta(x-y)$ is applied. It is nonzero only along one diagonal. What happens to the degeneracies in part (a)? Write the corresponding eigenfunctions and calculate the eigenvalues. You may work to lowest order in V_o

(c) Suppose that instead of the potential in (b), an infinite wall is erected along the diagonal, so that the particle is confined to $y \leq x$. What are the eigenfunctions and eigenvalues?



QM Problem 4

A beam of atoms in the state ℓ=1, s=1/2, j=3/2, moving in the x-direction, passes through a Stern-Gerlach apparatus in which $B = B_z \ll E_o/\mu_B$ where E_o is the spin-orbit energy (i.e. the separation between states of j=1/2 and j=3/2 for fixed ℓ=1, s=1/2.

The four emerging beams are separated, and each passes through a separate Stern-Gerlach apparatus in which $B = B_z \gg E_o/\mu_B$.

Into how many beams is each of the four beams further split, and what are the relative numbers of atoms in each of these beams?

Note: $H = H_o + \frac{\mu_B B}{\hbar}(L_z + 2S_z) + \frac{2W}{\hbar^2}\vec{L}\cdot\vec{S}$